Star Formation 2020

Q&A 19.05.2020

Worked Example - Driving Turbulence with Protostellar Outflows

Consider a collapsing protostellar core that delivers mass to an accretion disk at its center at a constant rate $\dot{M}_d$. A fraction f of the mass that reaches the disk is ejected into an outflow, and the remainder goes onto a protostar at the center of the disk. The material ejected into the outflow is launched at a velocity equal to the escape speed from the stellar surface. The protostar has a constant radius R_* as it grows.

a) momentum per unit stellar mass ejected by the outflow

Compute the momentum per unit stellar mass ejected by the outflow in the process of forming a star of final mass M_*. Evaluate this numerically for $f = 0.1$, $M_* = 0.5\,M_\odot$ and $R_* = 3\,R_\odot$.

The escape speed at the stellar surface, and thus the launch velocity of the wind, is

$$v_w = \sqrt{2\,G\,M_*(t)/R_*}$$

with the star's instantaneus mass $M_*(t)$. The momentum flux associated with the wind is therefore

$$\dot{p}_w = f\,\dot{M}_d\,v_w$$

The accretion rate onto the star is

$$\dot{M}_* = (1-f)\,\dot{M}_d$$

d. Thus at a time t after the star has started accreting, we have

$$M_*(t) = (1-f)\,\dot{M}_d\,t$$

and

$$\dot{p}_w =$$

$$f\,\dot{M}_d\,v_w = f\,\dot{M}_d\,\sqrt{2\,G\,M_*(t)/R_*} == f\,\dot{M}_d\,\sqrt{2\,G\,(1-f)\,\dot{M}_d\,t/R_*} = f\,(1-f)^{1/2}\,\dot{M}_d^{3/2}\,t^{1/2}\,\sqrt{\frac{2\,G}{R_*}}$$

The time required to accrete up to the star’s final mass is

$$t_f = M_*/\dot{M}_* = (1-f)^{-1}\,M_*/\dot{M}_d$$

where M_* is the final mass. To obtain the wind momentum per unit stellar mass, we must integrate $\dot{p}_w$ over the full time it takes to build up the star, then divide by the star’s mass. Thus we have

$$\langle p_w\rangle = \frac{1}{M_*}\int_0^{(1-f)^{-1}M_*/\dot{M}_d} f\,(1-f)^{1/2}\,\dot{M}_d^{3/2}\,t^{1/2}\,\sqrt{\frac{2\,G}{R_*}}\,dt =$$

$$\frac{2\sqrt{2-2f}\, f\, \dot{M}_d^{3/2}\left(-\frac{M_*}{(-1+f)\,\dot{M}_d}\right)^{3/2}\sqrt{\frac{G}{R_*}}}{3\,M_*} = \frac{2}{3}\frac{f}{1-f}\sqrt{\frac{2\,G\,M_*}{R_*}}$$

Evaluating numerically for the given values of f, M_*, and R_* gives

$$\langle p_w\rangle = \frac{2}{3}\frac{f}{1-f}\sqrt{\frac{2\,G\,M_*}{R_*}} = \frac{2}{3}\frac{0.1}{1-0.1}\sqrt{\frac{2\times 6.67\times 10^{-8}\times 0.5\times 2\times 10^{33}}{3\times 6.96\times 10^{10}}} = 1.87231\times 10^{6}$$

The result is $\langle p_w\rangle = 19\ \mathrm{km}\, s^{-1}\, M_\odot^{-1}$

b) Kinetic energy injection rate

The material ejected into the outflow will shock and radiate energy as it interacts with the surrounding gas, so on large scales the outflow will conserve momentum rather than energy. The terminal velocity of the outflow material will be roughly the turbulent velocity dispersion σ in the ambient cloud. If this cloud is forming a cluster of stars, all of mass M_*, with a constant star formation rate $\dot{M}_{\mathrm{cluster}}$, compute the rate at which outflows inject kinetic energy into the cloud.

Each outflow carries momentum $\langle p_w\rangle\, M_+$, and thus when it decelerates to terminal velocity σ the mass it has swept-up must be $M_w = (\langle p_w\rangle/\sigma)\, M_*$. The associated kinetic energy of a single outflow is

$$\mathcal{T}_w = \frac{1}{2} M_w\, \sigma^2 = \frac{1}{2} M_*\, \langle p_w\rangle\, \sigma$$

If the total star formation rate is $\dot{M}_{\mathrm{cluster}}$, then the rate at which new stars form is $\dot{M}_{\mathrm{cluster}}/M_*$ The rate of kinetic energy injection is therefore

$$\dot{\mathcal{T}} =$$

$$\frac{\dot{M}_{\mathrm{cluster}}}{M_*}\mathcal{T}_w = \frac{1}{2}\dot{M}_{\mathrm{cluster}}\,\langle p_w\rangle\,\sigma = \frac{1}{2}\dot{M}_{\mathrm{cluster}}\,\sigma\,\frac{2}{3}\frac{f}{1-f}\sqrt{\frac{2\,G\,M_*}{R_*}} = \frac{1}{3}\dot{M}_{\mathrm{cluster}}\,\sigma\,\frac{f}{1-f}\sqrt{\frac{2\,G\,M_*}{R_*}}$$

c) Balanced star formation rate

Suppose the cloud obeys Larson's relations, so its velocity dispersion, mass M, and size L are related by $\sigma = \sigma_1(L/\mathrm{pc})^{0.5}$ and $M = M_1(L/\mathrm{pc})^2$, where $\sigma_1 \approx 1\ \mathrm{km}\, s^{-1}$ and $M_1 \approx 100\, M_\odot$ are the velocity dispersion and mass of a 1 pc-sized cloud. Assuming the turbulence in the cloud decays exponentially on a timescale $t_{cr} = L/\sigma$, what star formation rate is required for energy injected by outflows to balance the energy lost via the decay of turbulence? Evaluate this numerically for $L = 1,\ 10$ and 100 pc.

The decay time is L/σ, to the decay rate must be the cloud kinetic energy $(3/2)\, M\, \sigma^2$ divided by this time. Thus

$$\dot{\mathcal{T}}_{\mathrm{dec}} = -\frac{3}{2}\frac{M\,\sigma^3}{L}$$

If we now set $\dot{\mathcal{T}}_w = -\dot{\mathcal{T}}_{\mathrm{dec}}$, we can solve for $\dot{M}_{\mathrm{cluster}}$. Doing so gives

$$\dot{\mathcal{T}} = \frac{1}{3}\dot{M}_{\mathrm{cluster}}\,\sigma\,\frac{f}{1-f}\sqrt{\frac{2\,G\,M_*}{R_*}} = \frac{3}{2}\frac{M\,\sigma^3}{L} \Rightarrow \dot{M}_{\mathrm{cluster}} = \frac{9}{2}\frac{1-f}{f}\sqrt{\frac{R_*}{2\,G\,M_*}}\,\frac{\sigma^2}{L}\,M$$

Using the Larson relations to evaluate this, note that $\sigma^2/L = \sigma_1/\mathrm{pc} \equiv a_c = 3.2\times 10^{-9}\ \mathrm{cm}\, s^{-1}$ is constant, and we are left with

$$\dot{M}_{\text{cluster}} = \frac{9}{2}\frac{1-f}{f}\sqrt{\frac{R_*}{2\,G\,M_*}}\,a_c\,M_1\left(\frac{L}{pc}\right)^2$$

Out[∘]//TableForm=

	L=1pc	L=10pc	L=100pc
$\dot{M}_{\text{cluster}}\,[M_\odot\ yr^{-1}]$	0.0000166907	0.00166907	0.166907

d) SF efficiency

If stars do form at the rate required to maintain the turbulence, what fraction of the cloud mass must be converted into stars per cloud free-fall time? Assume the cloud density is $r = M/L^3$. Again, evaluate numerically for $L = 1,$ 10 and 100 pc. Are these numbers reasonable? Conversely, for what size clouds, if any, is it reasonable to neglect the energy injected by protostellar outflows?

The mass converted into stars in 1 free-fall time is $\dot{M}_{\text{cluster}}\,t_{\text{ff}}$, so the quantity we want to compute is

$$f = \frac{\dot{M}_{\text{cluster}}}{M}\,t_{ff} \equiv \frac{t_{ff}}{t_*}$$

where t_* is the star formation timescale. From the previous part, we have

$$t_*^{-1} = \frac{\dot{M}_{\text{cluster}}}{M} = \frac{9}{2}\frac{1-f}{f}\sqrt{\frac{R_*}{2\,G\,M_*}}\,a_c =$$

$$\frac{9}{2}\frac{1-0.1}{0.1}\sqrt{\frac{3\times 6.96\times 10^{10}}{2\times 6.67\times 10^{-8}\times 0.5\times 2\times 10^{33}}}\,3.2\times 10^{-9} = 5.12734\times 10^{-15}\ s^{-1}$$

and in units of Myr^{-1}

```
In[∘]:= 5.12734256112564`*^-15 × 365 × 24 × 3600 × 10^6
Out[∘]= 0.161696
```

The SILCC project - feedback simulations

https://hera.ph1.uni-koeln.de/~silcc/

```
In[∘]:= videoSN = Import[
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```

Only Supernovae

```
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Out[∘]=

Supernovae + Wind

In[∘]:= `videoWSN`

Out[∘]= 

Supernovae + Wind + Radiation

In[∘]:= `videoRWSN`

Out[∘]= 

Massive star formation feedback