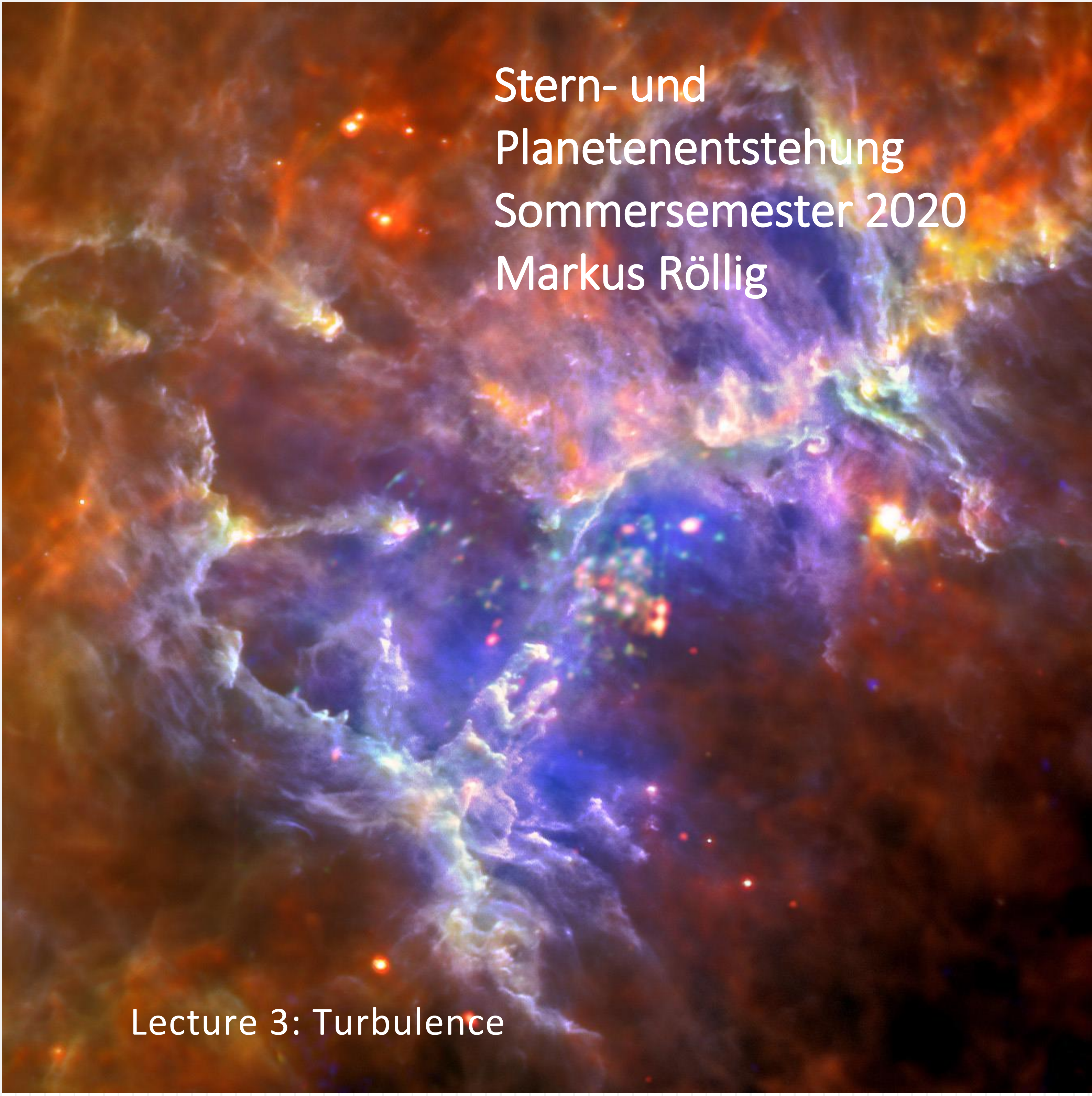


Stern- und Planetenentstehung

Sommersemester 2020

Markus Röllig

Lecture 3: Turbulence

http://exp-astro.physik.uni-frankfurt.de/star_formation/index.php

VORLESUNG/LECTURE

Raum: Physik - 02.201a

dienstags, 12:00 - 14:00 Uhr

SPRECHSTUNDE:

Raum: GSC, 1/34, Tel.: 47433, (roellig@ph1.uni-koeln.de)

dienstags: 14:00-16:00 Uhr

Nr.	Thema	Termin
1	Observing the cold ISM	21.04.2020
2	Observing Young Stars	28.04.2020
3	Gas Flows and Turbulence Magnetic Fields and Magnetized Turbulence	05.05.2020
4	Gravitational Instability and Collapse	12.05.2020
5	Stellar Feedback	19.05.2020
6	Giant Molecular Clouds	26.05.2020
7	Star Formation Rate at Galactic Scales	02.06.2020
8	Stellar Clustering	09.06.2020
9	Initial Mass Function – Observations and Theory	16.06.2020
10	Massive Star Formation	23.06.2020
11	Protostellar disks and outflows – observations and theory	30.06.2020
12	Protostar Formation and Evolution	07.07.2020
13	Late Stage stars and disks – planet formation	14.07.2020

3 MAGNETIC FIELDS AND MAGNETIZED TURBULENCE

Real SF regions are highly magnetized!

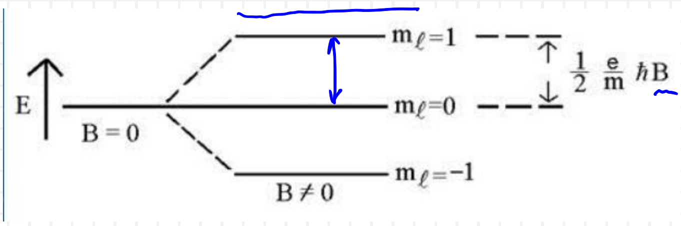
3.1 OBSERVING MAGNETIC FIELDS

3.1.1 Zeeman Measurements

Most direct measurement of magnetic fields: Zeeman effect:

Slight shift in energy levels in the presence of magnetic fields

No B Field: level energy depends on rel. orientation of electron spin to orbital angular momentum vector (not on the orientation of the net angular momentum vector!)



With B Field: states with different orientation of net angular momentum vector have different energies

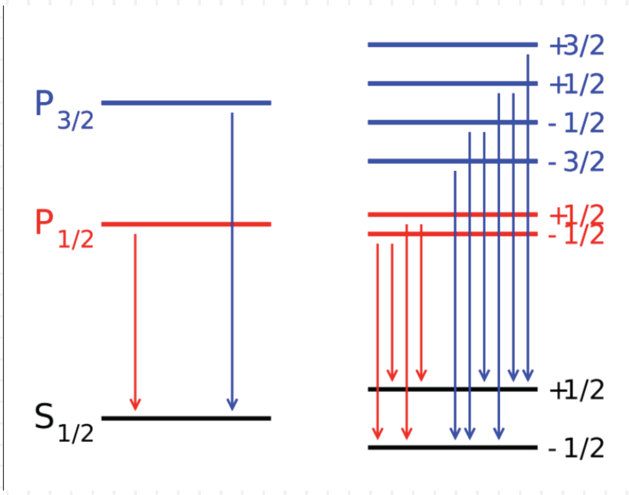


Abbildung 1Zeeman splitting of the Lyman alpha transition

Energy levels split into sublevels Zeeman splitting (anomalous, i.e. incl. spin)

$\Delta\nu = B\,Z$ *B*: magn. field strength, *Z*: Zeeman sensitivity

(OH: 0.98 Hz/μG)

Z largest for molecules/atoms with unpaired electrons in the outer shell (H, OH, Cn, CH, CCS, SO, O_2, …)

Example OH: $\sigma_\nu = \nu_0 \left(\frac{\sigma_v}{c}\right)$, $\nu_0 = 1.667\ \text{GHz}$

$$\sigma_v \sim 0.1\,\frac{\text{km}}{\text{s}} \quad => \quad \left(\frac{\sigma_\text{v}}{c}\right) \sim 10^{-6}\,, \quad \text{so} \quad \sigma_\nu \sim 1\text{kHz}$$

unless B>>1000µG (hardly ever) , Zeeman splitting smaller than Doppler line width

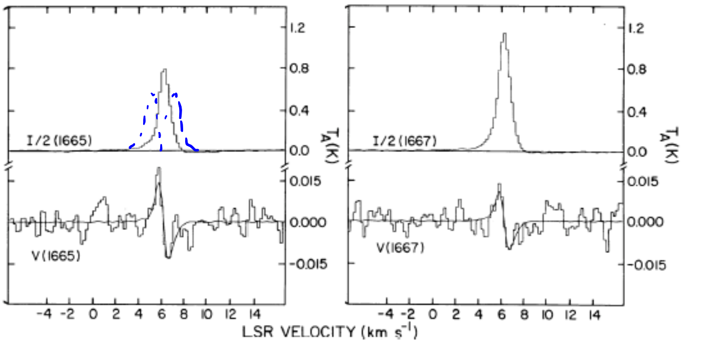


Figure 5: One of the first observations of the Zeeman effect in molecular clouds, using the OH 1665 MHz and OH 1667 MHz lines (Goodman et al., 1989)

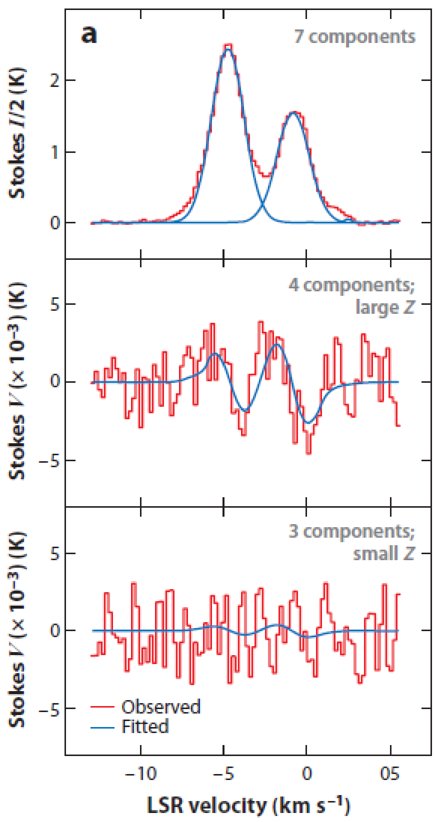


Abbildung 2Zeeman splitting of CN in DR21(OH) Crutcher 2012

Trick: different Zeeman sublevels emit radiation with different polarization (right & left circ. polarized)

Stokes I: total intensity

Stokes V: circularly polarized radiation

By measuring the energy split we can measure the strength of the radiation field (and sometimes also the orientation)

Energy in magn. Fields?

GMCs: $n \sim 100\text{ cm}^{-3}$ $(\rho \sim 10^{-22}\text{g cm}^{-3})$, v~few km s^{-1}

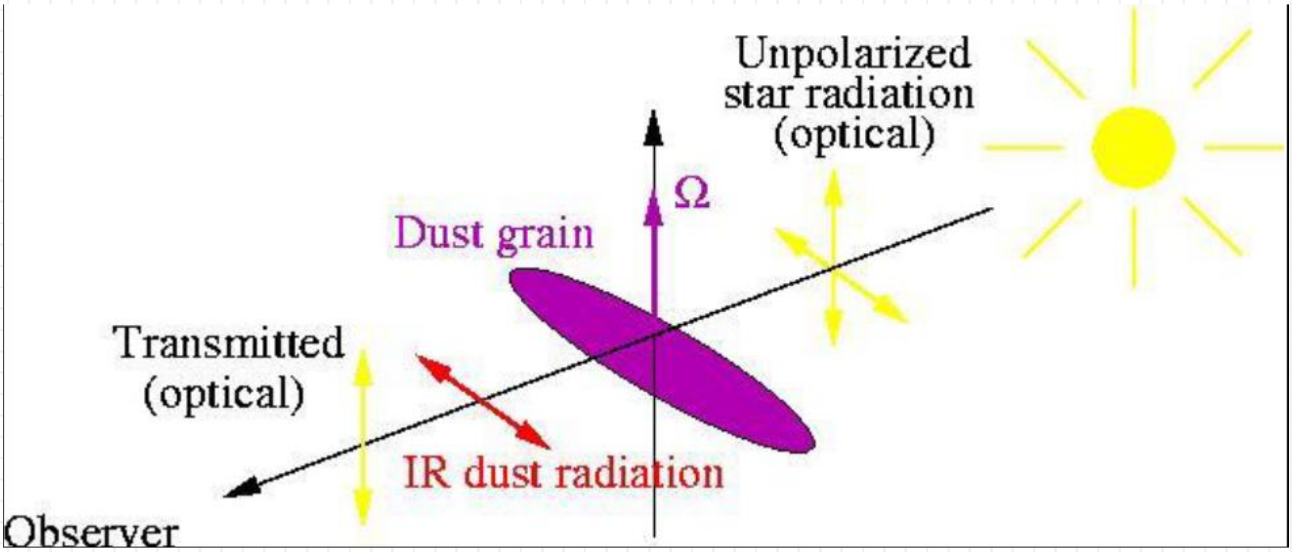
$$E_k \sim \rho\, v^2 \sim 10^{-22}\text{g cm}^{-3} \times (3 \times 10^5\text{cm s}^{-1})^2 \sim 10^{11}\text{erg cm}^{-3}$$

The energy density then is:

$$E_B = \frac{B^2}{8\pi} \sim \frac{(10\mu G)^2}{8\pi} \sim 10^{-11}\text{erg cm}^{-3}$$

Comparable to kinetic energy density => dynamically significant in the flow!

3.1.2 Dust grain alignment in magnetic fields



optical polarization: parallel to magnetic field
IR polarization: perpendicular to magnetic field

- Paramagnetic dissipation of energy (very slow) (Davis & Greenstein 1951)
- Radiative torque alignment (Hoang & Lazarian 2008)

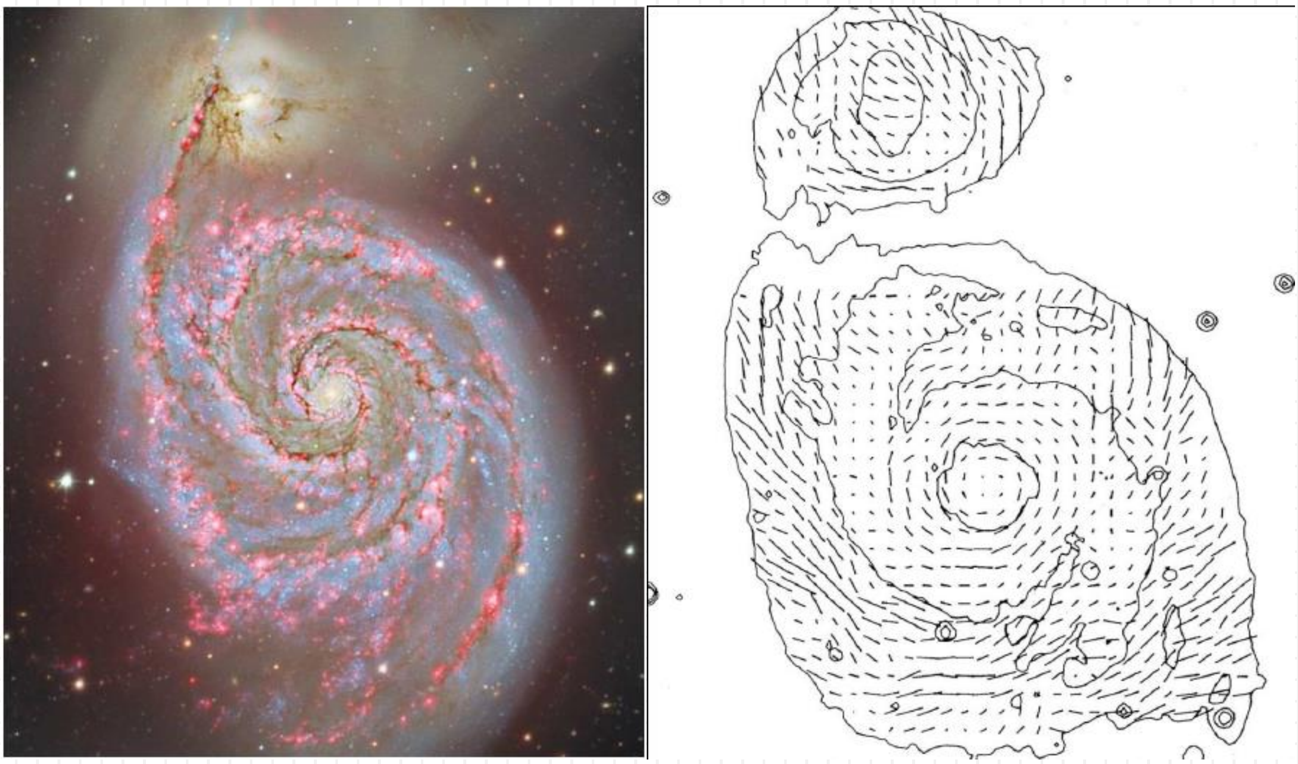


Abbildung 3 M51 Scarott et al(1977)

3.2 Equations and Characteristic Numbers for Magnetized Turbulence

magnetic field in a plasma: MHD evolution equation

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{B} \times \vec{v}) = -\nabla \times (\eta\, \nabla \times \vec{B})$$

$\vec{B}$: magnetic field, $\vec{v}$:fluid velocity, (ions which carry all mass), η: electrical resistivity (spezifischer elektr. Widerstand $\eta = R\frac{A}{l}$ Ohm m

(if $\eta = const.,=> \quad \nabla \cdot \vec{B} = 0$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{B} \times \vec{v}) = \eta \nabla^2 \vec{B}$$

compare:

- momentum equation: $\nu\nabla^2\vec{v}$: describes diffusion of momentum
- $\eta\nabla^2\vec{B}$: describes diffusion of the magnetic field (scalar simplified)

Again: dimensional analysis: *B* is the characteristic magn. field strength, *L/V* characteristic time scale

$$\frac{B\,V}{L} + \frac{B\,V}{L} \sim \eta\,\frac{B}{L^2}$$
$$1 \sim \frac{\eta}{V\,L}$$

➔ analogous: magnetic Reynolds number

$$Rm = \frac{L\,V}{\eta}$$

magnetic diffusion is significant only if $Rm{\sim}1$ or smaller

typical cloud: *L*: few 10 pc, *V*: few km s^{-1} , *B*: few 10 µG
η: depends on ionization fraction in the gas and ion-neutral collision rate ~10^{22}-10^{23} cm^2 s^{-1}.
LV ~ 10^{25} $cm^2 s^{-1}$ ➔ *Rm* ~ 10^2-10^3

on large scales magn. diffusion is unimportant but important on smaller scales large Rm ➔ $\eta = 0$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{B} \times \vec{\mathrm{v}}) = 0$$

To understand what this means: magnetic flux ϕ threading some fluid elements:

$$\phi = \int_A \vec{B} \cdot \hat{\mathrm{n}} \, dA$$

using Stokes's theorem this can be written as (*C* is the boundary curve of *A*):

$$\phi = \oint_C \vec{B} \cdot d\vec{\mathrm{l}}$$

time derivative:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \int_A \frac{\partial \vec{B}}{\partial t} \hat{\mathrm{n}} \, dA + \oint_C \vec{B} \cdot \vec{\mathrm{v}} \times d\vec{\mathrm{l}} \\ &= \int_A \frac{\partial \vec{B}}{\partial t} \hat{\mathrm{n}} \, dA + \oint_C \vec{B} \times \vec{\mathrm{v}} \cdot d\vec{\mathrm{l}} \end{aligned}$$

second term on the right:
$\vec{\mathrm{v}} \times d\vec{\mathrm{l}}$: area swept out by unit $d\vec{\mathrm{l}}$ per unit time
$\vec{B} \cdot \vec{\mathrm{v}} \times d\vec{\mathrm{l}}$: flux crossing this area $(\nabla \cdot B = 0, \text{ allows to exchange } \cdot \leftrightarrow \times)$

apply Stokes's theorem again

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \int_A \frac{\partial \vec{B}}{\partial t} \hat{\mathrm{n}} \, dA + \int_A \nabla \times (\vec{B} \times \vec{\mathrm{v}}) \cdot \hat{\mathrm{n}} \, dA \\ &= \int_A \left[\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{B} \times \vec{\mathrm{v}}) \right] \cdot \hat{\mathrm{n}} \, dA \\ &= 0 \end{aligned}$$

Therefore, if *Rm* is large, the magnetic flux through each fluid element is conserved – called flux-freezing (plasma and magnetic field are coupled, moving the plasma takes the magn. field lines along and vice versa).

On large scales, the magnetic field lines move with the plasma!

The momentum equation including magnetic forces:

$$\frac{\partial}{\partial t}(\rho\,\vec{v}) = -\nabla\cdot(\rho\,\vec{v}\,\vec{v}) - \nabla P + \rho\,\nu\,\nabla^2\,\vec{v} + \frac{1}{4\pi}(\nabla\times\vec{B})\times\vec{B}$$

dimensional analysis:

$$\frac{\rho V^2}{L} \sim -\frac{\rho V^2}{L} + \frac{\rho c_s^2}{L} + \frac{\rho \nu V}{L^2} + \frac{B^2}{L}$$

$$1 \sim 1 + \frac{c_s^2}{V^2} + \frac{\nu}{VL} + \frac{B^2}{\rho V^2}$$

already defined: ($\mathcal{M} = V/c_s$, $Re = LV/\nu$) now Alfvénic Mach number

$$\mathcal{M}_A = V/\mathrm{v}_A$$

with the Alfvén speed (magneto-acoustic speed of wave in MHD, analogous to sound speed in HD)

$$\mathrm{v}_A = \frac{B}{\sqrt{4\pi\rho}}$$

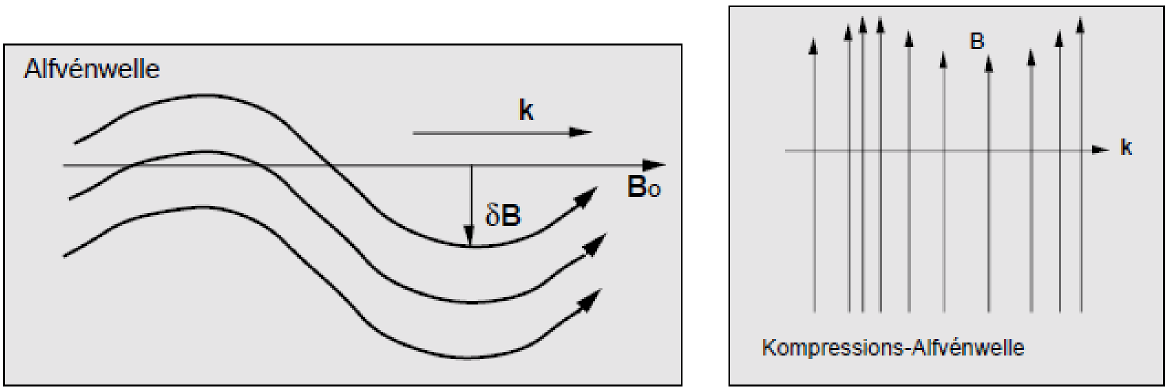


Abbildung 6.2: Links: Shear-Alfvénwelle mit paralleler Ausbreitung. Die Amplitude steht senkrecht auf dem homogenen Magnetfeld. Rechts: Kompressionswelle (senkrechte Ausbreitung). Die Magnetfeldlinien werden nicht "verbogen" sondern komprimiert, daher der Name Kompressionswelle.

Abbildung 4Alfvén waves (H.Wobig, MHD LEcture Notes , MPI for Plasmaphysics (http://edoc.mpg.de/display.epl?mode=doc&id=628457&col=33&grp=1312)

If $\mathcal{M}_A \gg 1$, then the magnetic force term is unimportant, while if $\mathcal{M}_A \ll 1$ it is dominant.

Typically: $n \sim 100\ cm^{-3}, B \sim 10\mu G, V \sim km\ s^{-1}$, then $\mathrm{v}_A \sim few\ km\ s^{-1}$

Thus, the flows in molecular clouds are highly supersonic ($\mathcal{M} \gg 1$) but only trans-Alfvénic ($\mathcal{M}_A \sim$)

⇨ magnetic forces have a significant influence

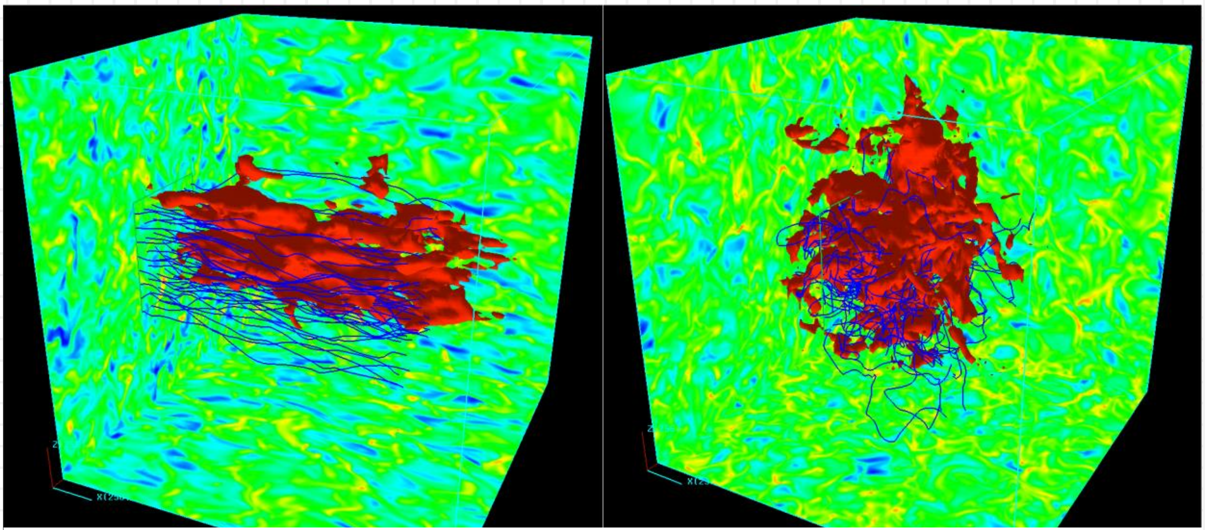


Abbildung 5 strong B field, sub-Alfvénic turbulence (left), and Alfvénic, weak B fields (right) (Jim Stone, Princeton, http://www.astro.princeton.edu/~jstone/turb.html

3.3 Non-Ideal MHD

3.3.1 Ion-Neutral Drift

Most gas in a MC is neutral, not ionized, ionization fraction $\leq 10^{-6}$

Only the few ion feel the Lorentz force directly. Most particles in a MC feel the effect of magnetic field indirectly (collisions with ions)

If the collisional coupling is strong then the gas behaves as a perfect plasma, however if ionization fraction is low, then the coupling is imperfect and ions and neutral move with different speed.

The field follows the ions (because of their low resistivity) and flux freezing for them is very good approximation, but the neutrals are able to drift across field lines.

This is called Ion-Neutral-Drift or ambipolar diffusion.

Forces acing on ions and neutrals:

Ions feel the Lorentz force: $\vec{F}_L = \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B}$

drag force due to ion-neutral (*i*: ions, *n*: neutrals) collisions:

$$\vec{F}_d = \gamma \rho_n \rho_i (\vec{\mathrm{v}}_i - \vec{\mathrm{v}}_n)$$

In a very weakly ionized fluid, ions and neutral quickly reach terminal velocity => $\vec{F}_L \approx \vec{F}_d$. With $\vec{v}_d = (\vec{v}_i - \vec{v}_n)$ we get:

$$\vec{v}_d = \frac{1}{4\pi\gamma\rho_n\rho_i}(\nabla \times \vec{B}) \times \vec{B}$$

now assuming that the field is frozen into the ions:

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{B} \times \vec{v}_i) = 0$$

eliminating $\vec{v}_i$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{B} \times \vec{v}_n) = \nabla \times \left(\frac{\vec{B}}{4\pi\gamma\rho_n\rho_i} \times [\vec{B} \times (\nabla \times \vec{B})]\right)$$

comparison with the MHD evolution equation:

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{B} \times \vec{v}) = -\nabla \times (\eta\, \nabla \times \vec{B})$$

shows that the resistivity, produced by the ambipolar drift is not a scalar and non-linear (it depends on $\vec{B}$)

Magnitude of the resistivity produced by the ambipolar drift is:

$$\eta_{AD} = \frac{B^2}{4\pi\rho_i\rho_n\gamma}$$

γ : ion-neutral drag coefficient ($cm^3s^{-1}g^{-1}$). At low speeds, ions induce a dipole moment in the nearby neutrals -> Coulomb interaction and enhances the collision cross section.
Therefore the magnetic Reynolds number is

$$RM = \frac{LV}{\eta_{AD}} = \frac{4\pi\rho_i\rho_n\gamma}{B^2} \approx \frac{4\pi LV\rho^2 x\gamma}{B^2}$$

where $x = n_i/n_n$ is the ion fraction (assumed to be <<1)

Ion neutral drift will allow the magnetic field lines to drift through the fluid on large scale lengths L such that $Rm \lesssim 1$.

Characteristic length scale for AD: $$L_{AD} = \frac{B^2}{4\pi\rho^2 x\gamma V}$$

Estimate of L_{AD}

Smith & Mac Low 1997: $\gamma \approx 9.2\ 10^{13}\mathrm{cm}^3\mathrm{s}^{-1}\mathrm{g}^{-1}$

Tielens (2005): $$n_i \approx 2 \times 10^3\mathrm{cm}^{-3}\left(\frac{n_H}{10^4\mathrm{cm}^{-3}}\right)^{\frac{1}{2}}\left(\frac{\zeta}{10^{-16}\mathrm{s}^{-1}}\right)^{1/2}$$

(ζ: ionization rate of hydrogen due to cosmic rays)

$$n_H \sim 100\ \mathrm{cm}^{-3} \quad => x \approx 10^{-6}$$

$$Rm \approx 50$$

$$L_{AD} \approx 0.5\ \mathrm{pc}$$

for cores:

$$L_{AD} \approx 0.05\ \mathrm{pc}$$

Thus, we expect the gas to act like a fully ionized gas on large scales (> L_{AD}) and to switch over to behaving hydro-dynamically on smaller scales.

3.3.2 Turbulent Reconnection

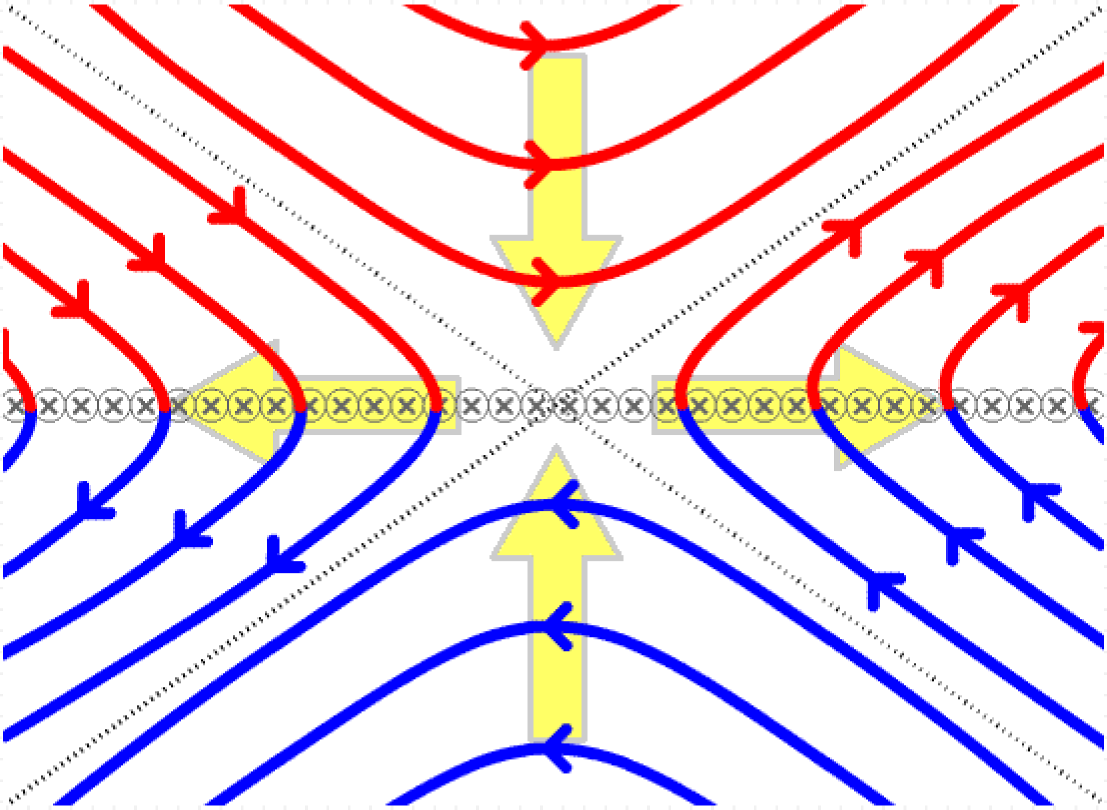


Abbildung 6 Image Wikipedia: https://en.wikipedia.org/wiki/Magnetic_reconnection

Fluid elements with non-zero resistivity and opposing field line orientations are brought into contact. Then the field lines can break and re-orientate. Field lines may slip out of the material -> reduction of magn. pressure and energy density. The released energy is transformed into heat and (probably kin. energy of accelerated material (e.g. in solar flares).

Sweet-Parker model

Two regions of opposing field lines meet at a plane. In that plane a large current needs to flow in order to maintain the magnetic fields. Within this sheet reconnection can occur.

Analog to ion-neutral drift we define a Reynolds-like number :

Lundqvist number $R_L = \frac{LV}{\eta}$ η: true microphysical resistivity

reconnection rate depends on geometry: matter flows in, it reconnects and then it needs to exit for the new matter to be able to reconnect.

Exit is only possible at the Alfvén speed. Max speed for new matter to enter the reconnection zone: $\sim R_L^{-1/2} v_A$

With the electrical conductivity σ one finds for a plasma:

$$\eta = \frac{c^2}{4\pi\sigma} = \frac{n_e e^2}{m_e m_{H_2} < \sigma\mathrm{v} >_{e-H_2}} \approx 10^{17} x\ \mathrm{s}^{-1}$$

$< \sigma\mathrm{v} >_{e-H_2} \approx 10^{-9} cm^3 s^{-1}$ is the mean cross-section times the velocity for electron-ion collisions.

$$\eta \approx \frac{10^3 cm^2 s^{-1}}{x}$$

With our typical values for $x \approx 10^{-6}$ we find $\eta \approx 10^9 cm^2 s^{-1}$ and using $L \approx 10pc\ and\ V \approx$ ^3 ... km/s this implies

$$R_L \sim 10^{16}$$

This makes the reconnection speed tiny $\sim 10^{-8} \times \mathrm{v}_A$

This would mean, that it is very unimportant, but it would also prohibit any solar flares! So the theoretical result is wrong! and the true reconnection speed has to be much larger.