Differences between Nuclear Reactions in Stars and in the Laboratory

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Remarks:

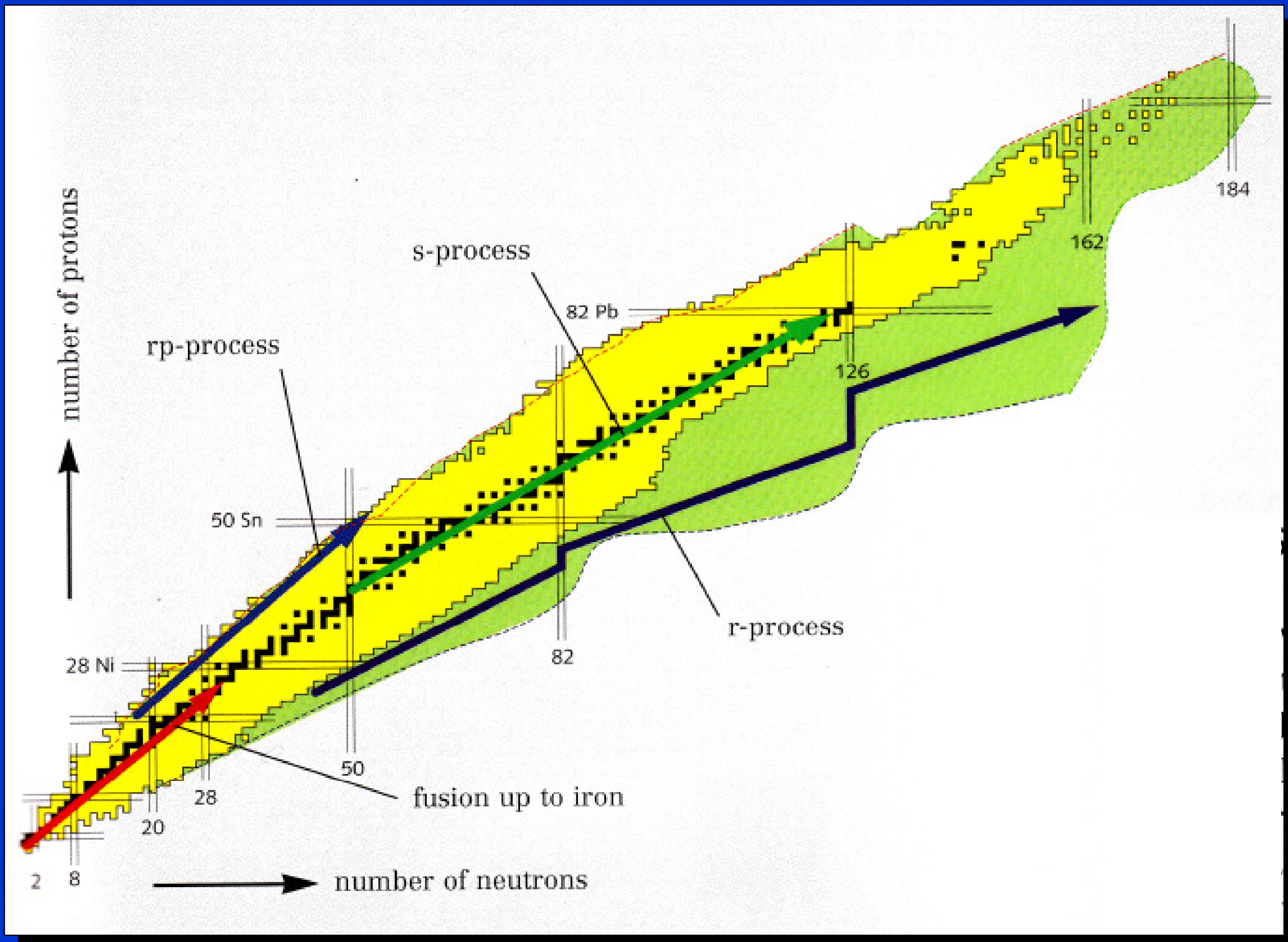
1. This is converted from an animated, non-linear (internally hyperlinked) presentation. To save space some slides were left out. On the other hand, some slides are still included which I did not have time to show.
2. The hyperlink structure of my presentations results in a dynamic, non-linear sequence which cannot well represented in a linear PDF. I tried to sort the slides into „subsections“ and within those to approximate the shown sequence of slides.
3. Since I often use animation effects putting several layers on top of each other, some of the slides may appear overloaded or inaccessible in the PDF version. Where easily possible (and important) I tried to distribute the content of one slide to several, although then each slide appears incomplete.
4. Some of the „pictures“ are actually animations which obviously appear static in the PDF. Some external movies could not be included at all.

Outline

- Astrophysical burning close to stability
  - < Fe: Hydrostatic burning (cases known, challenge is low energy)
  - > Fe: s-Process, γ-Process
- Definitions
  - Astrophysical reaction rates
  - Reaction mechanisms
- Stellar effects
  - (Modification of decay half-lives)
  - Stellar vs Laboratory Rates
- Summary

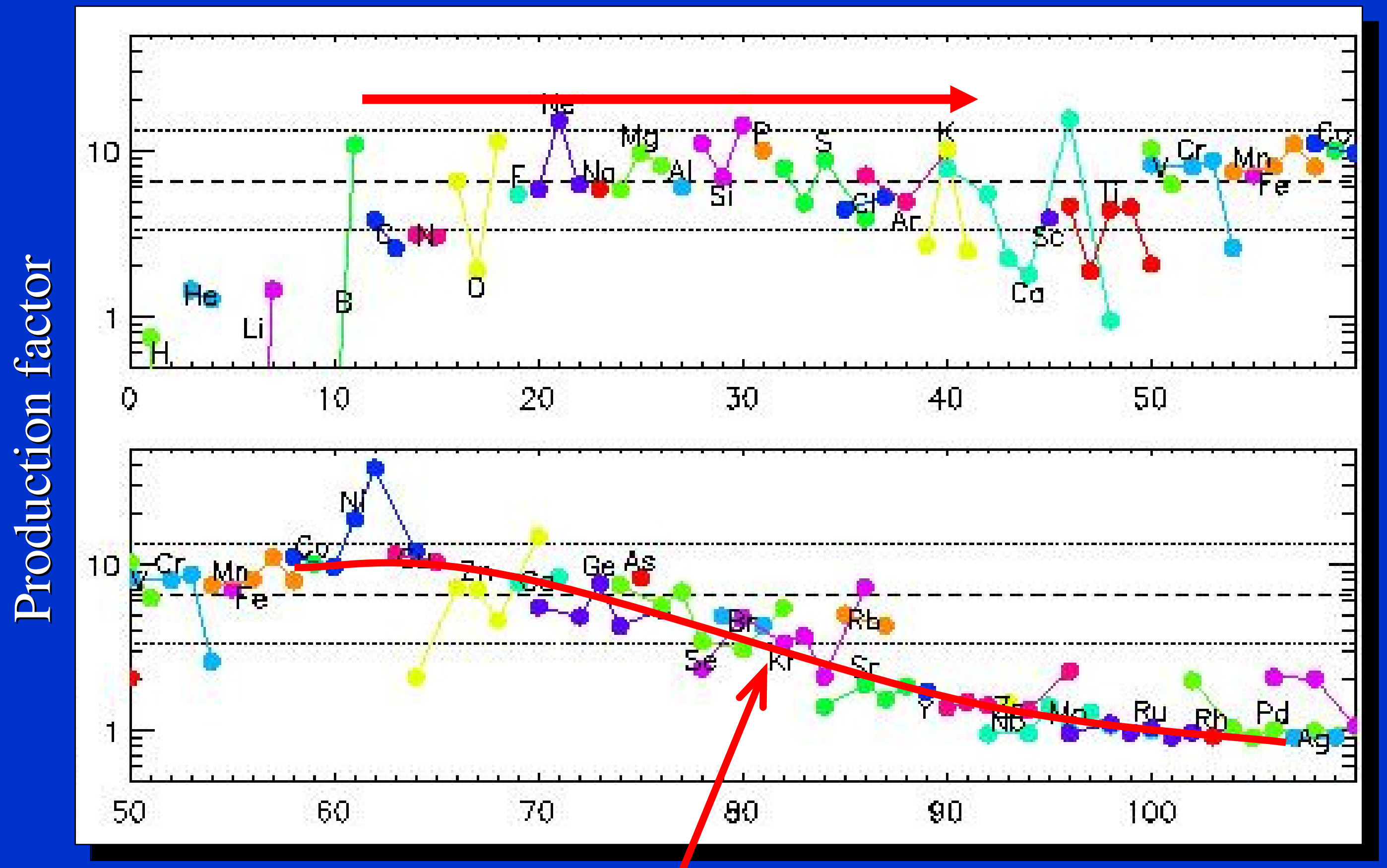
Astrophysical Burning Close to Stability

Nucleosynthesis Processes



Nucleosynthesis Results (15 $M_\odot$)

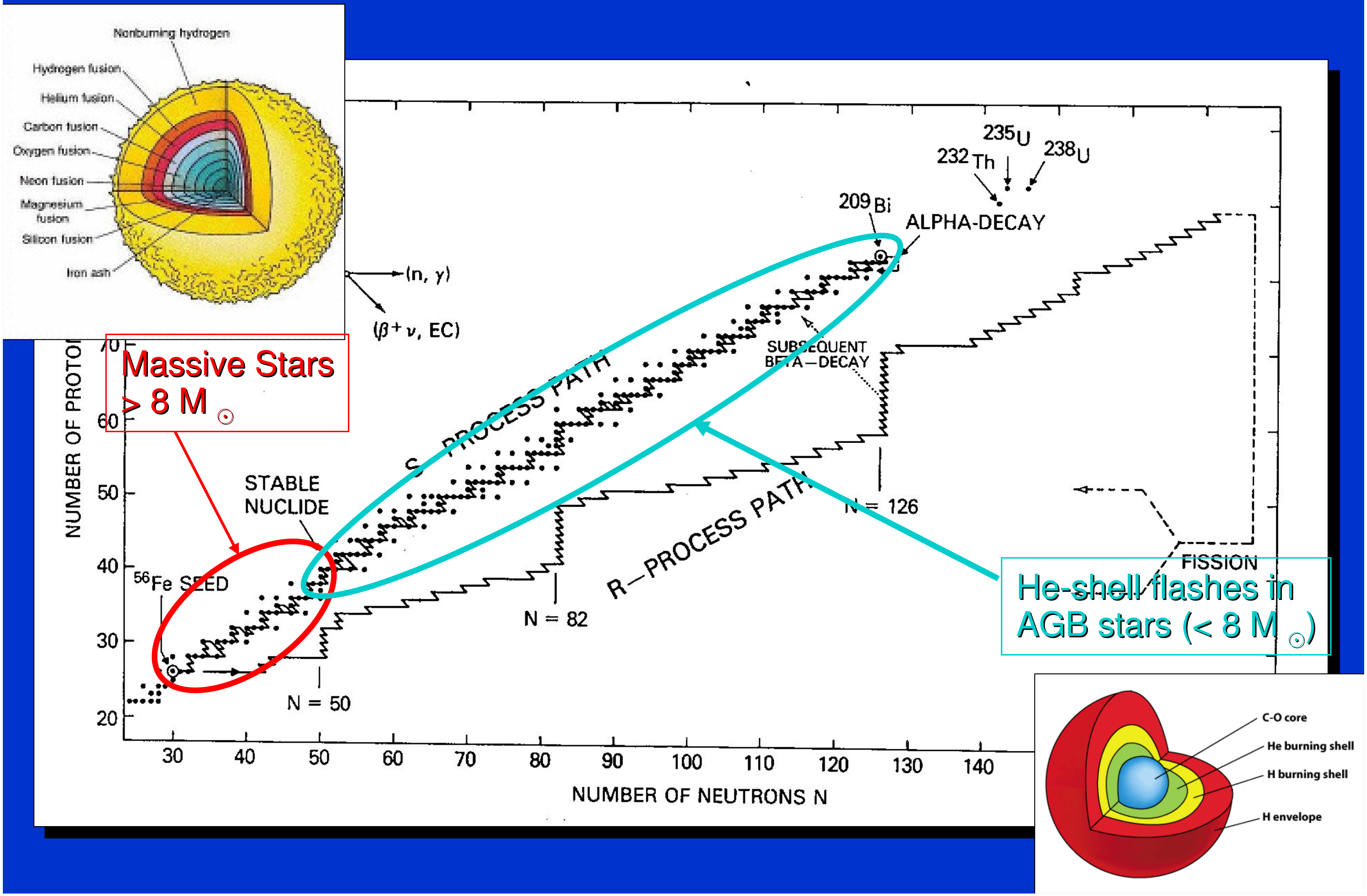
Mostly hydrostatic burning



„weak“ s-process component

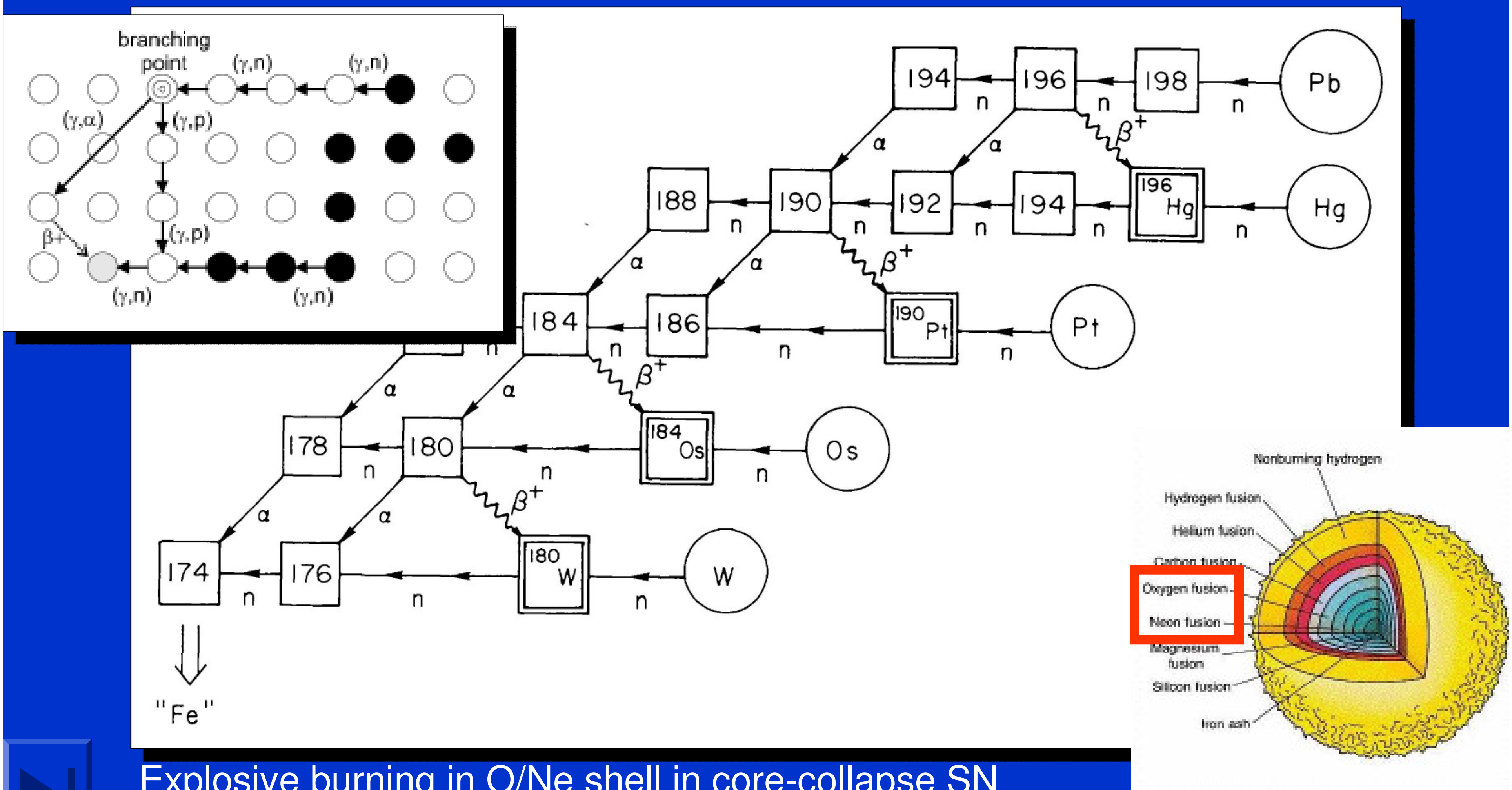
Rauscher et al. 2002 (with UCSC and LLNL)

s-Process Path



The γ-Process

Photodisintegration of seed nuclei (produced in situ or inherited from prestellar cloud).
NOT total disintegration, of course! (just the right amount)

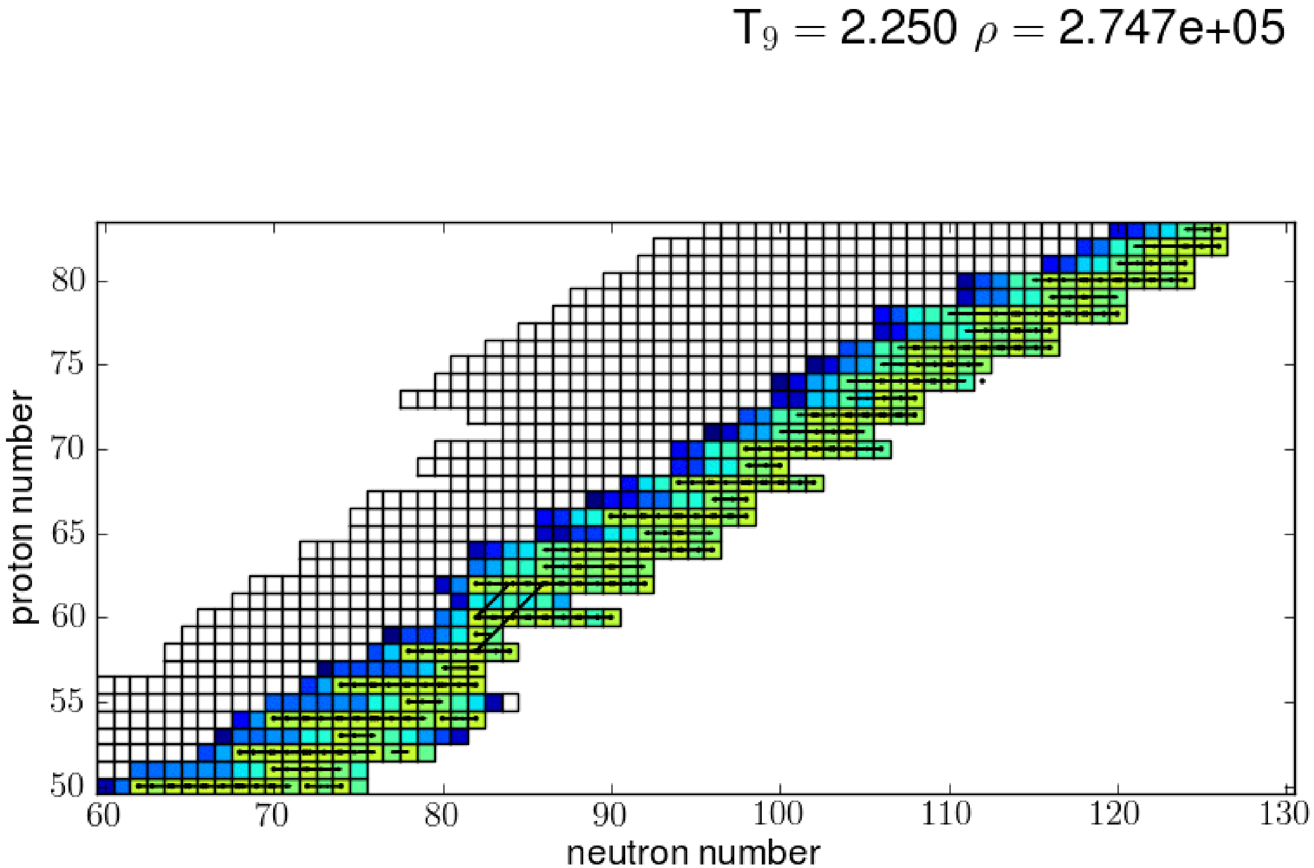


Explosive burning in O/Ne shell in core-collapse SN

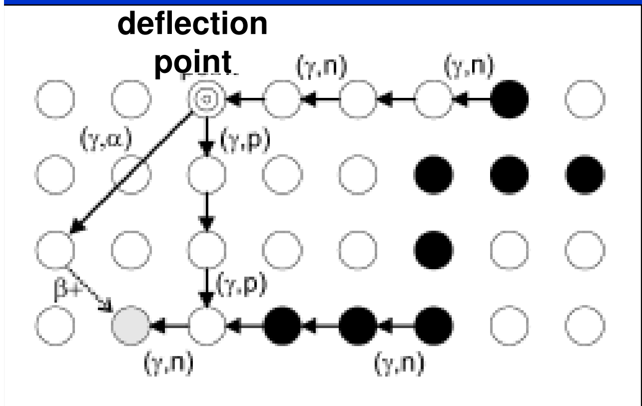
Woosley & Howard 1978; Prantzos et al 1990; Rayet et al 1995

Photodisintegration of stable seed nuclei

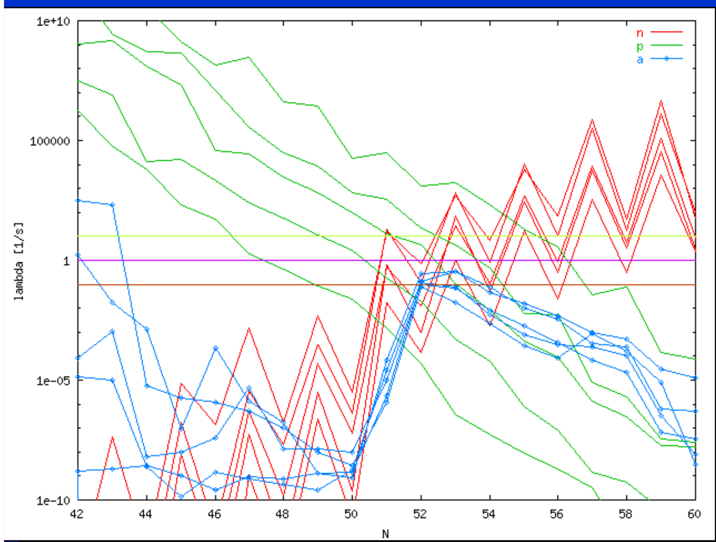
- Not an equilibrium process!
- Competition of (γ,n), (γ,p), (γ,α) rates determine path and destruction speed at each temperature.
- Strong nuclear constraints on required astrophysical conditions for each group of nuclei, e.g., at high T all heavier nuclei are destroyed.



γ-Process Path Deflections

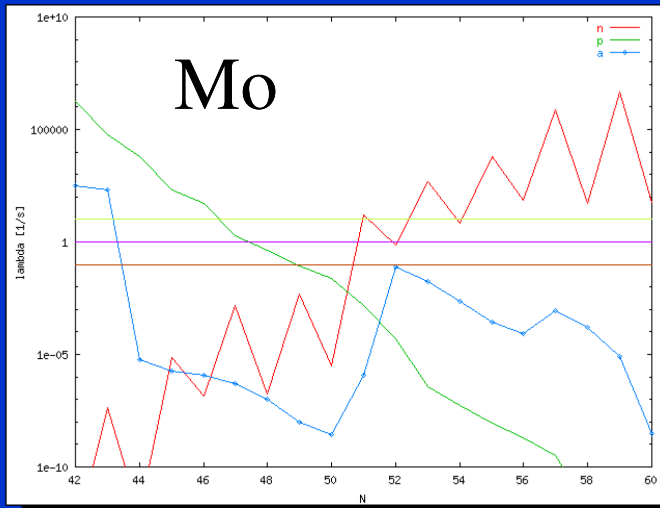


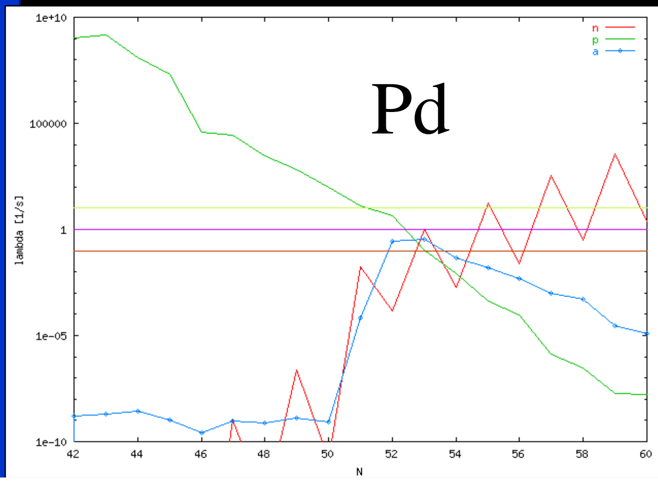
- (γ,n) determine timescale
- (γ,p/α) determine flow to lower mass



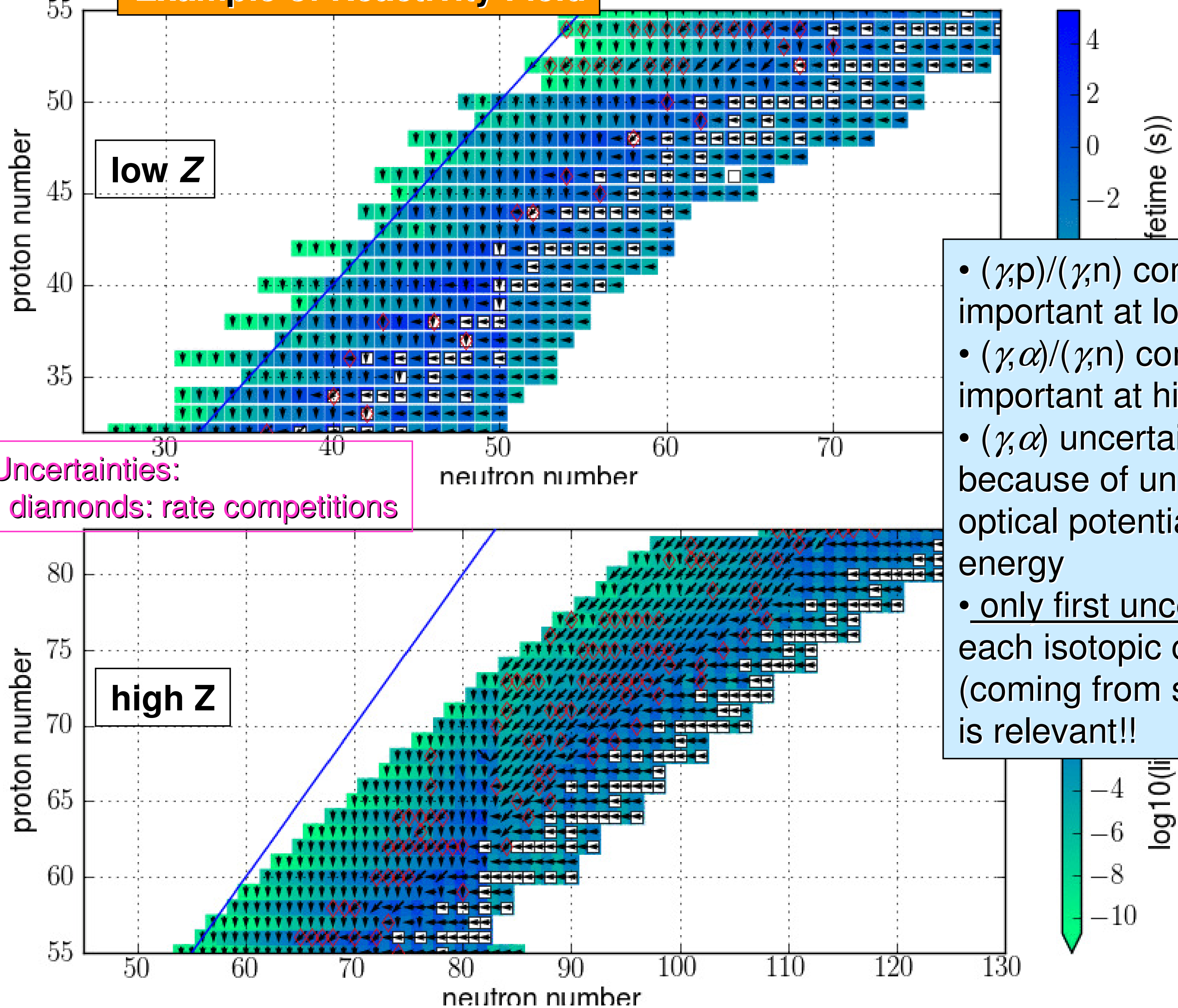
- quick change in dominating reaction within isotopic chain
- mostly only competition between (γ,n) and one other particle channel
- primary targets for experimental investigation (but unstable!)

(γ,n), (γ,p), (γ,α) rates at T_9=2.5 for Z=42-46 (Mo-Pd)





Example of Reactivity Field



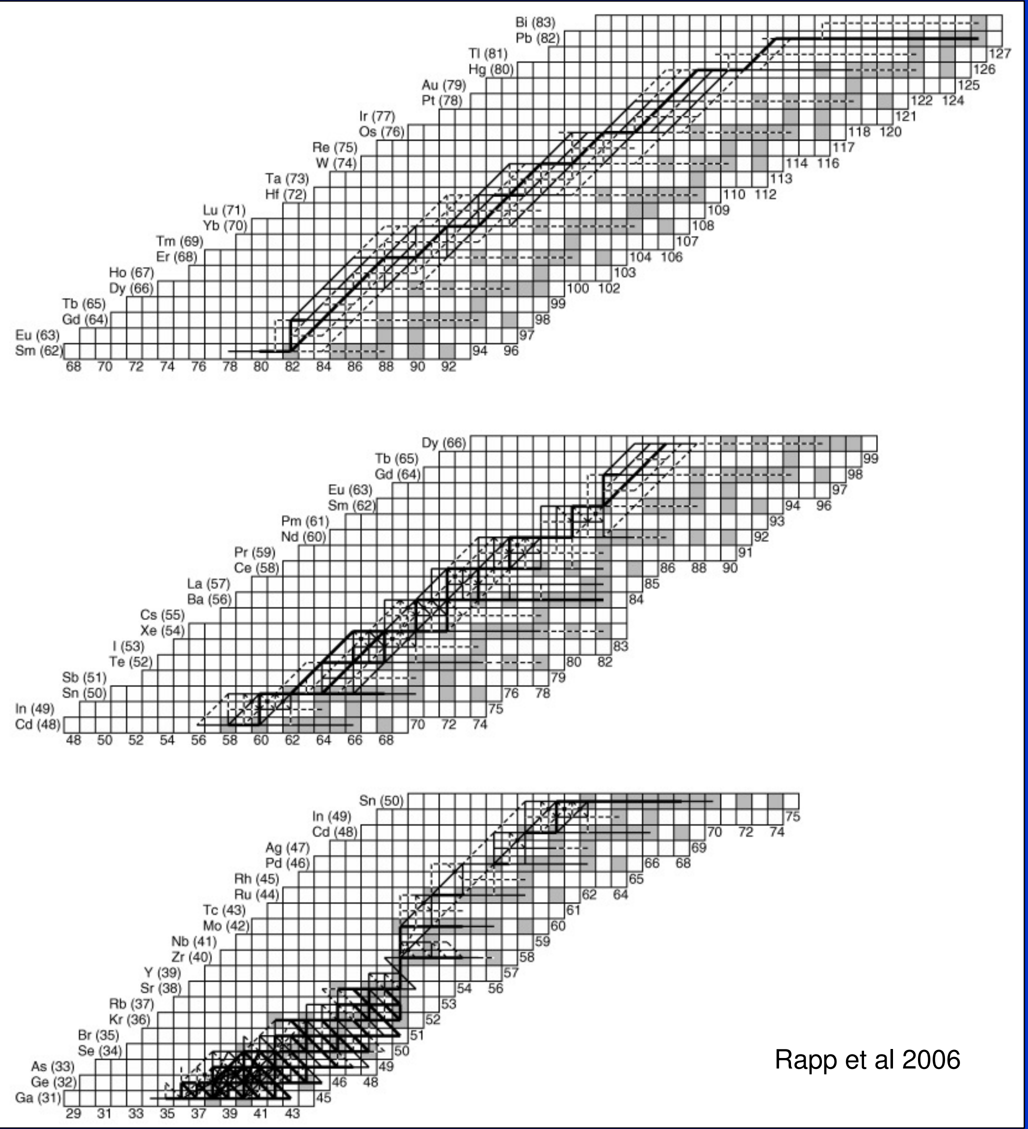
Uncertainties:
- diamonds: rate competitions

- (γ,p)/(γ,n) competition important at low *Z*
- (γ,α)/(γ,n) competition important at high Z
- (γ,α) uncertain because of uncertain optical potential at low energy
- <u>only first uncertainty</u> in each isotopic chain (coming from stability) is relevant!!

Relevant Nuclear Input

General p-Process Properties:

- Temperatures of $2<T_9<3.5$ (depending on scenario)
- Starting from s- and r-nuclides (previously included in star or produced by star), dominant flows are (γ,n)
- With decreasing proton- and/or α-separation energy, (γ,p) and (γ,α) become faster: deflection of path ("branching")
- For "light" p-elements, (n,γ) can hinder efficient photodisintegration
- (n,p) reactions can speed up matter flow
- Some scenarios: proton captures in mass region of light p-nuclei



Rapp et al 2006

Nuclear Input for γ-Process Studies (Theory)

- Prediction should be possible with Hauser-Feshbach statistical model of compound reactions
- Largest uncertainty due to optical potentials
  - usually derived from scattering at much higher energy than astrophysically relevant
  - not well constrained at low energy (around and below Coulomb barrier)
  - imaginary part should be energy dependent
- Largest deviation with α-potentials
  - notorious example: ^{144}Sm(α,γ) factor 12 variation when fitting to exp data
  - usual deviation a factor of 2-3 (too high) with "standard" potential (McFadden & Satchler)
- "Standard" proton potential from Brueckner-Hartree-Fock calculation with Local Density Approximation
  - Jeukenne, Lejeune, Mahaux (1977) with low-energy modifications by Mahaux (1982)
  - Works well at higher energy but isovector imaginary part not constrained at low energy
  - Indication of a <u>possibly required modification</u> at astrophysical energies?
  - Usual deviation of factors 1.0-2.0 (but not always too low)

Astrophysical Reaction Rates

Reaction Networks

Reactions *i(j,k)m* lead to change in plasma composition:

- NN reactions:

$$\left(\frac{\partial n_i}{\partial t}\right)_\rho = \left(\frac{\partial n_j}{\partial t}\right)_\rho = -r_{ij} = -\frac{1}{1+\delta_{ij}} n_i n_j \left\langle \sigma^* v \right\rangle_{ij}$$

$$\left(\frac{\partial n_k}{\partial t}\right)_\rho = \left(\frac{\partial n_m}{\partial t}\right)_\rho = +r_{ij} = \frac{1}{1+\delta_{ij}} n_i n_j \left\langle \sigma^* v \right\rangle_{ij}$$

- Nγ, NL reactions, decays:

$$\left(\frac{\partial n_i}{\partial t}\right)_\rho = -r_i = -n_i \lambda_i \quad ; \quad \left(\frac{\partial n_m}{\partial t}\right)_\rho = +r_i = n_i \lambda_i$$

Reaction Rate (MB)

$$r_{12} = \frac{1}{1+\delta_{12}} n_1 n_2 \left\langle \sigma^* v \right\rangle_{12} = \frac{1}{1+\delta_{12}} \rho^2 Y_1 Y_2 N_A^2 \left\langle \sigma^* v \right\rangle_{12}$$

Number of reactions per time and volume

$$\left\langle \sigma v \right\rangle^*_{Aa} \propto \frac{1}{G_A^{\text{norm}}} \sum_\mu \left(\int \left\{ \frac{g_A^\mu}{g_A^0} \sigma_{Aa}^\mu E_A^\mu e^{-\left(E_A^\mu + \varepsilon_A^\mu\right)/(kT)} \right\} dE_A^\mu \right)$$

$$= \ldots = \frac{1}{G_A^{\text{norm}}} \int \left\{ \sum_\mu \frac{g_A^\mu}{g_A^0} E_A^\mu \sigma_{Aa}^\mu e^{-E_A^0/(kT)} \right\} dE_A^0 = \frac{1}{G_A^{\text{norm}}} \int \sigma_A^{\text{eff}} E_A^0 e^{-E_A^0/(kT)} dE_A^0$$

stellar reactivity

Nucleus-Photon Rate

With Planck distribution of photons:

$$r_{m\gamma} = n_m \lambda_{m\gamma}(T)$$

$$\lambda_{m\gamma}(T) = \frac{1}{\pi^2 c^2 \hbar^3} \int_0^\infty \frac{\sigma^*_{m\gamma}(E_\gamma) E_\gamma^2}{\mathrm{e}^{E_\gamma / kT} - 1} dE_\gamma$$

Connection to capture rate by detailed balance:

$$\lambda_{m\gamma} = \left(\frac{A_i A_j}{A_m} \right)^{3/2} \frac{(2J_i + 1)(2J_j + 1)}{2J_m + 1} \frac{G_i^{\mathrm{norm}}(T)}{G_m^{\mathrm{norm}}(T)} \left(\frac{\mu kT}{2\pi\hbar^2} \right)^{3/2} \mathrm{e}^{-Q_{ij}/kT} \left\langle \sigma^* v \right\rangle_{ij}$$

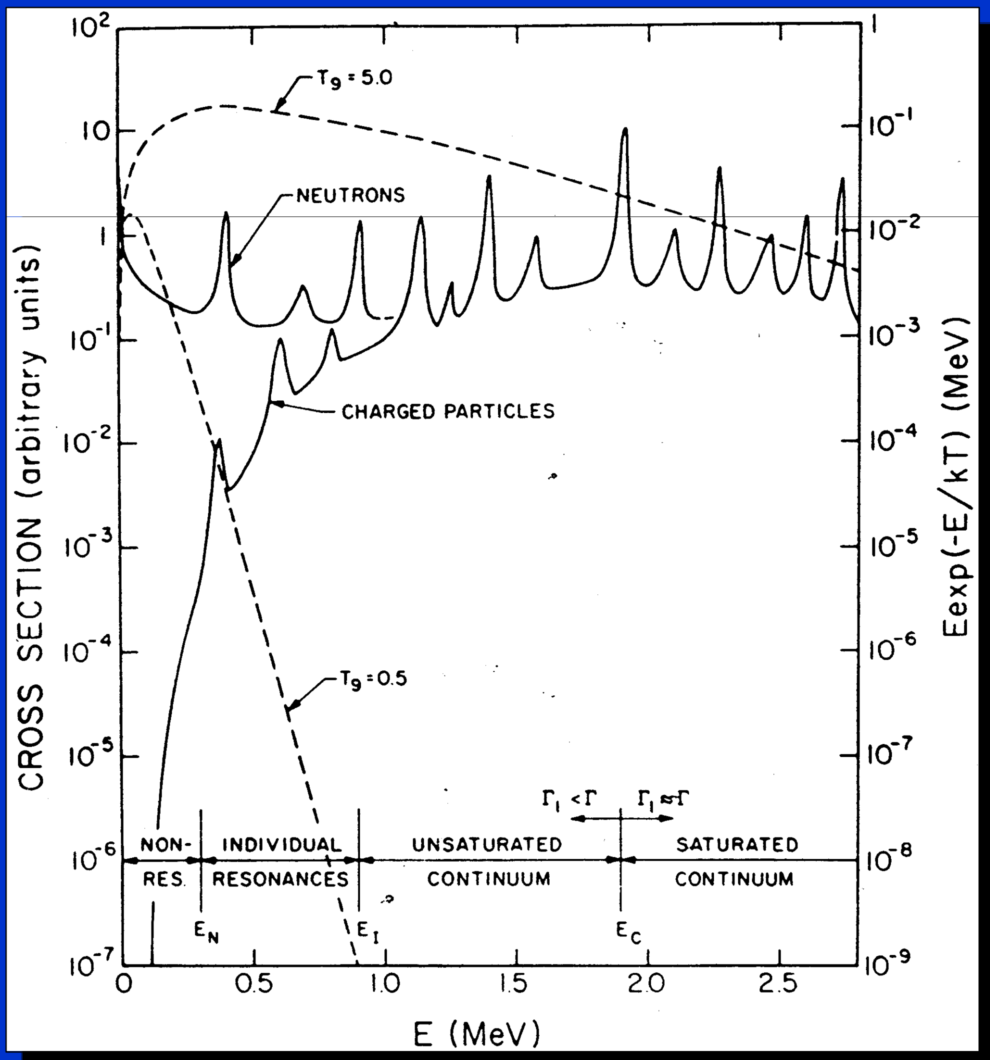
Nuclear Partition Functions

$$G_0(T) = \frac{1}{2J_0+1}\sum_{i=0}^{k}(2J_i+1)\,\mathrm{e}^{-E_i/kT} + \int_{E_k}^{E_{\max}}\sum_{\mathrm{J},\pi}(2\mathrm{J}+1)\,\mathrm{e}^{-\varepsilon_i/kT}\rho(\varepsilon,J,\pi)\,d\varepsilon$$

PF is proportional to number of different configurations at given temperature *T*. Corrections due to loss of nucleons to the continuum may apply at $T > 10$.

Reaction Mechanisms

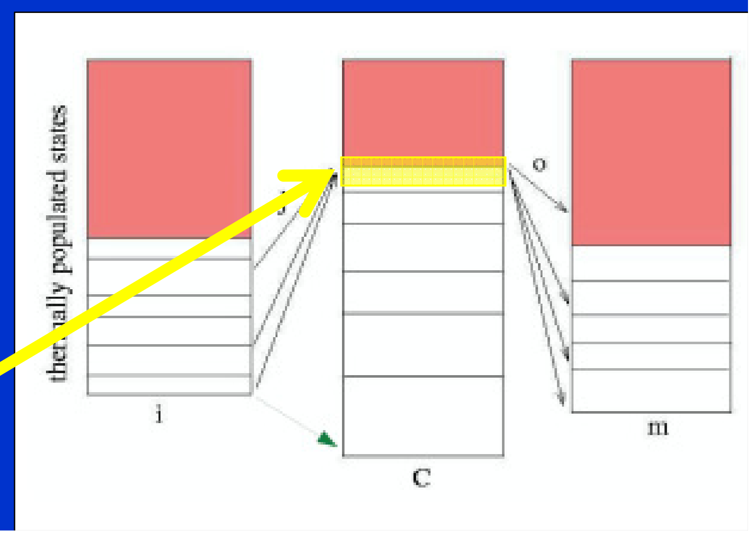
Reaction Mechanisms



Regimes:

1. Overlapping resonances: statistical model (Hauser-Feshbach)
2. Single resonances: Breit-Wigner, R-matrix
3. Without or in between resonances: Direct reactions

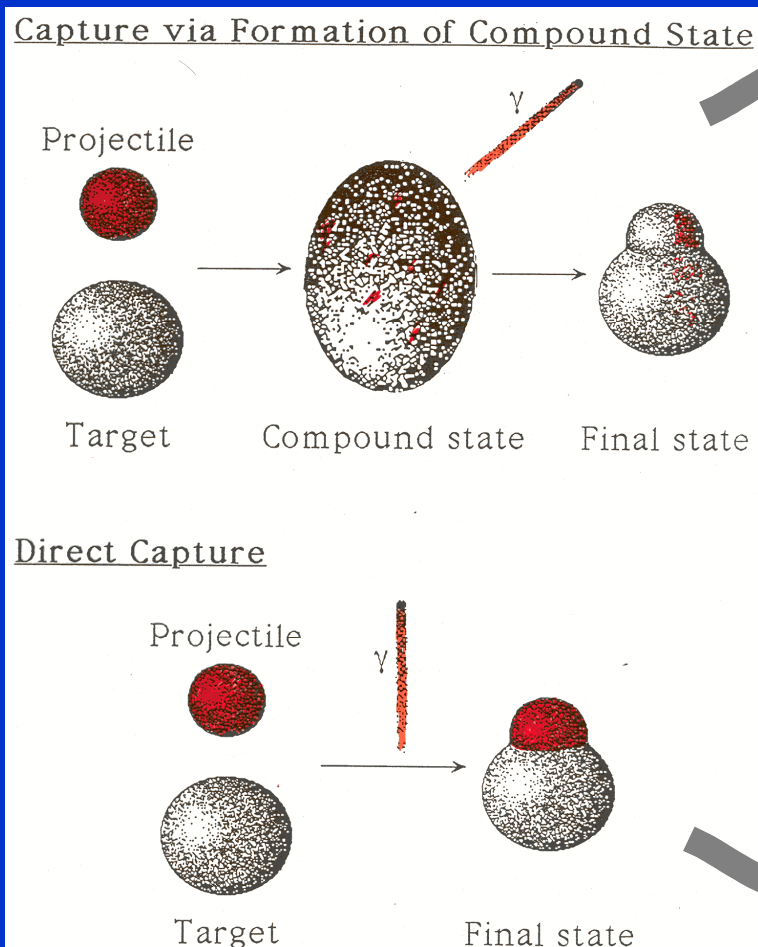
Determined by nucl. level density

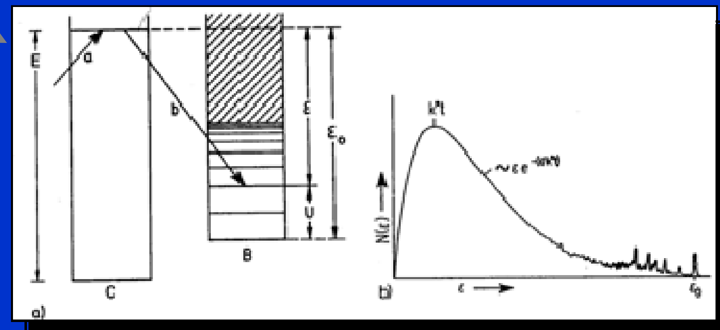


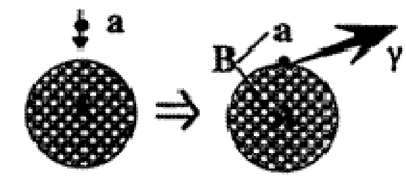
Reaction Mechanisms II

Statistical Model (Hauser-Feshbach):

$$\sigma^{\mathrm{CN}}_{\alpha\to\beta} = \sigma^{\mathrm{form}}_{\alpha} b_\beta = \sigma^{\mathrm{form}}_{\alpha} \frac{\langle \Gamma_\beta \rangle}{\langle \Gamma_{\mathrm{tot}} \rangle} \propto \frac{\langle \Gamma_\alpha \rangle \langle \Gamma_\beta \rangle}{\langle \Gamma_{\mathrm{tot}} \rangle}$$







A + a → B + γ

A ... target nucleus

a ... projectile

B = A ⊕ a ... residual nucleus

$$\frac{d\sigma}{d\Omega} = \left|\langle \phi_\beta | O_{EM} | \chi_\alpha \phi_\alpha \rangle\right|^2 \propto S \left| \int d\vec{R}\, \phi_{Aa} O_{EM} \chi_\alpha \right|^2$$

Hauser-Feshbach (statistical model) cross section is averaged Breit-Wigner cross section

$$\sigma_i(j,o)_{HF}$$

$$= \frac{\pi}{k_j^2}\sum_J (2J+1)\frac{(1+\delta_{ij})}{(2I_i+1)(2I_j+1)}W(j,o,J,\pi)\frac{T_j(E,J,\pi)T_o(E,J,\pi)}{T_{tot}(E,J,\pi)}$$

stat. model

$$= \langle\sigma_i(j,o)_{BW}\rangle \qquad \text{with}$$

$$\sigma_i(j,o)_{BW} = \frac{\pi}{k_j^2}\sum_n (2J_n+1)\frac{(1+\delta_{ij})}{(2I_i+1)(2I_j+1)}\frac{\Gamma_{j,n}\Gamma_{o,n}}{(E-E_n)^2+(\Gamma_n/2)^2}$$

Breit-Wigner

$$T_j(E,J,\pi) = \frac{2\pi}{D(E,J,\pi)}\langle\Gamma_j(E,J,\pi)\rangle$$

Transmission coeffs.

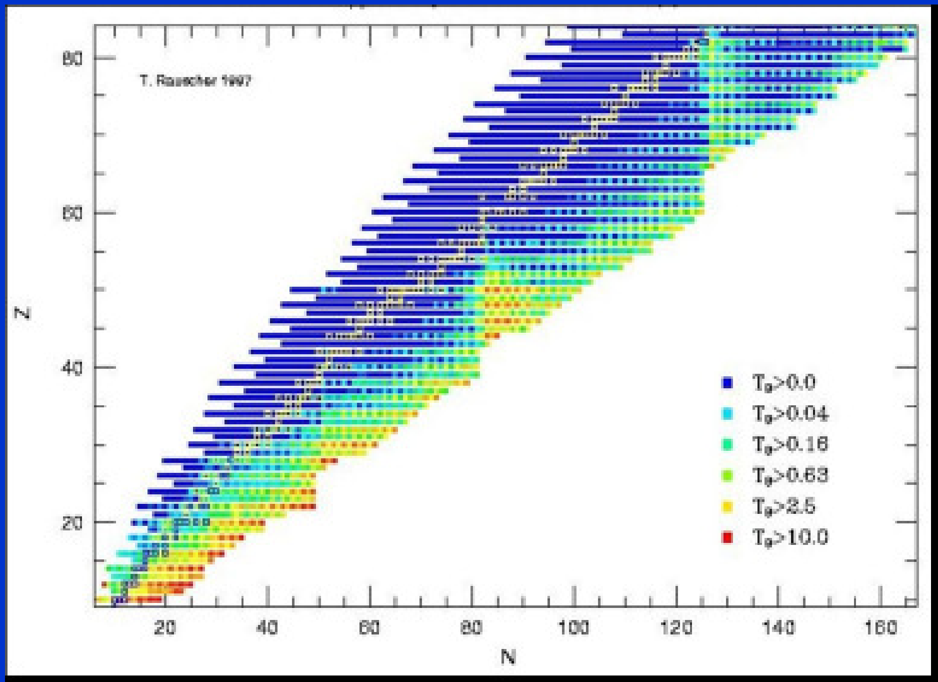
$$W(j,o,E,J,\pi) = \left\langle\frac{\Gamma_j(E,J,\pi)\Gamma_o(E,J,\pi)}{\Gamma_n(E,J,\pi)}\right\rangle \cdot \frac{\langle\Gamma(E,J,\pi)\rangle}{\langle\Gamma_j(E,J,\pi)\rangle\langle\Gamma_o(E,J,\pi)\rangle}$$

width fluctuation corrections

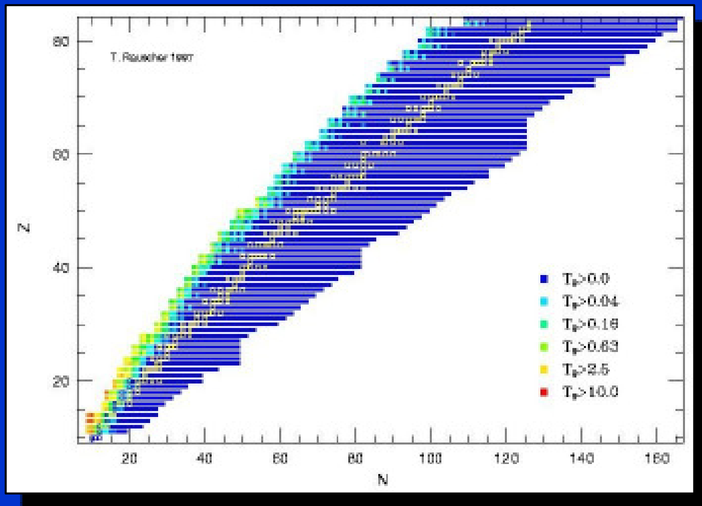
Reaction Mechanism Comparison

Applicability of statistical model

Neutron induced reactions

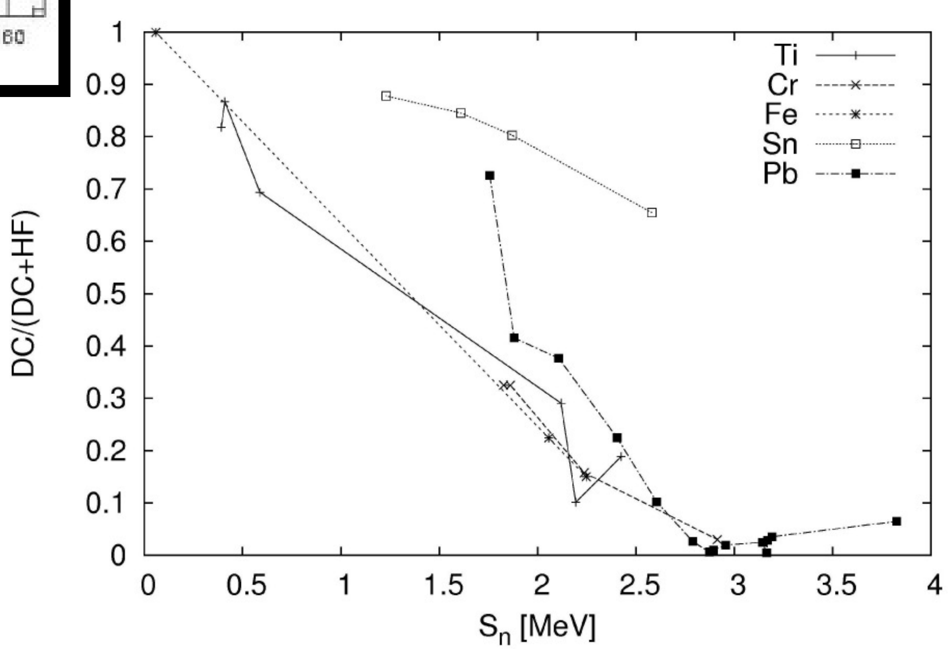


Rauscher et al., PRC 56 (1997) 1613



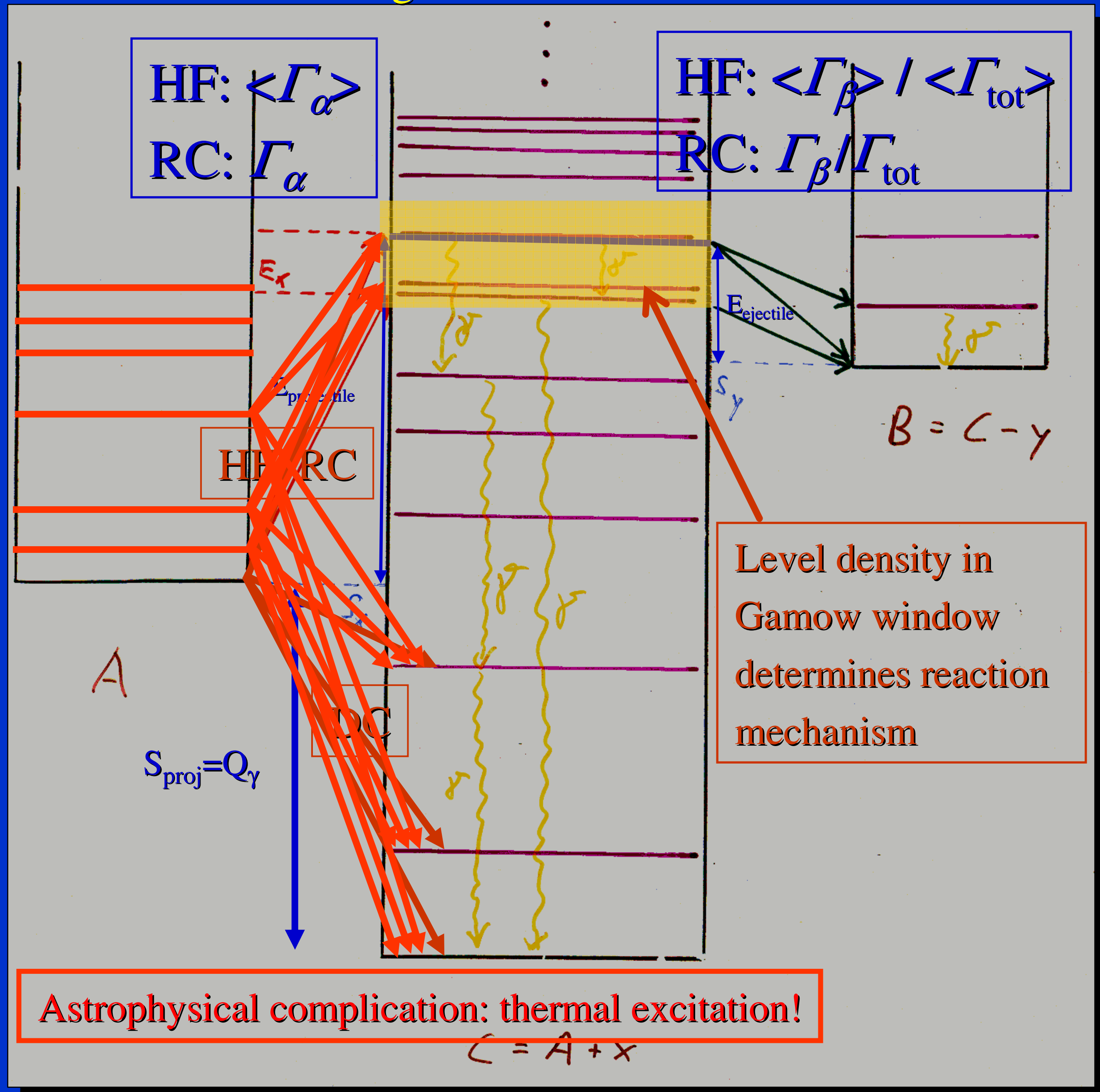
Proton-induced reactions

Comparison DC and Hauser-Feshbach



T. Rauscher; J. Phys. G 35 (2008) 014026

Energetics in Nuclear Reactions



Stellar vs Laboratory Rates

Stellar Effects

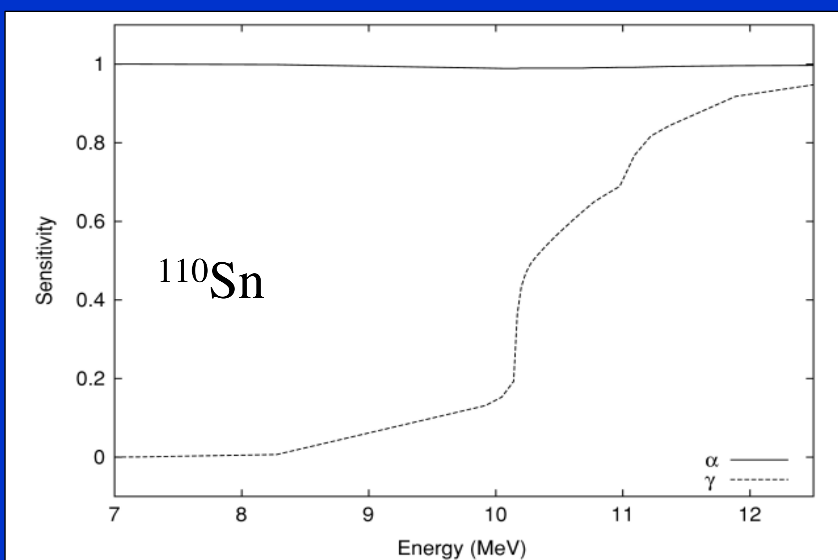
- Stellar effects act for all nuclei and environments but lead to larger deviations from laboratory rates > Fe
  - because of higher nuclear level density in target nuclei
  - because of higher Coulomb barriers
- Important effects:
  - Sensitivities of rates and cross sections
  - Relevant energy windows
  - Thermal population of target states and the *stellar* rate
    - Ground state contribution to stellar rate (+ implications for s- and γ-process)
    - Transitions from excited states, reciprocity, and Q-value rule
      - Exceptions
  - Photodisintegrations

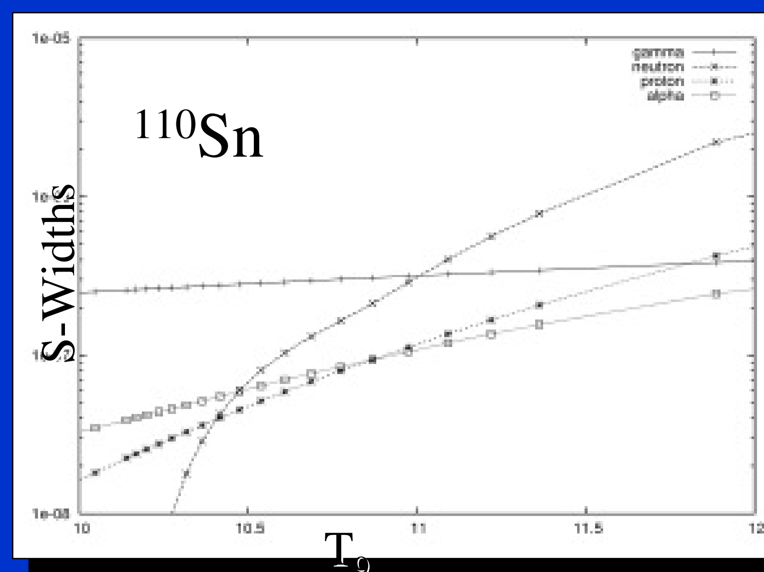
Sensitivities

Relative importance of widths

- Average widths (=transmission coefficients) determine the Hauser-Feshbach cross section
- γ-widths not necessarily the smallest ones at astrophysical energies!
- Similar for Breit-Wigner resonance widths

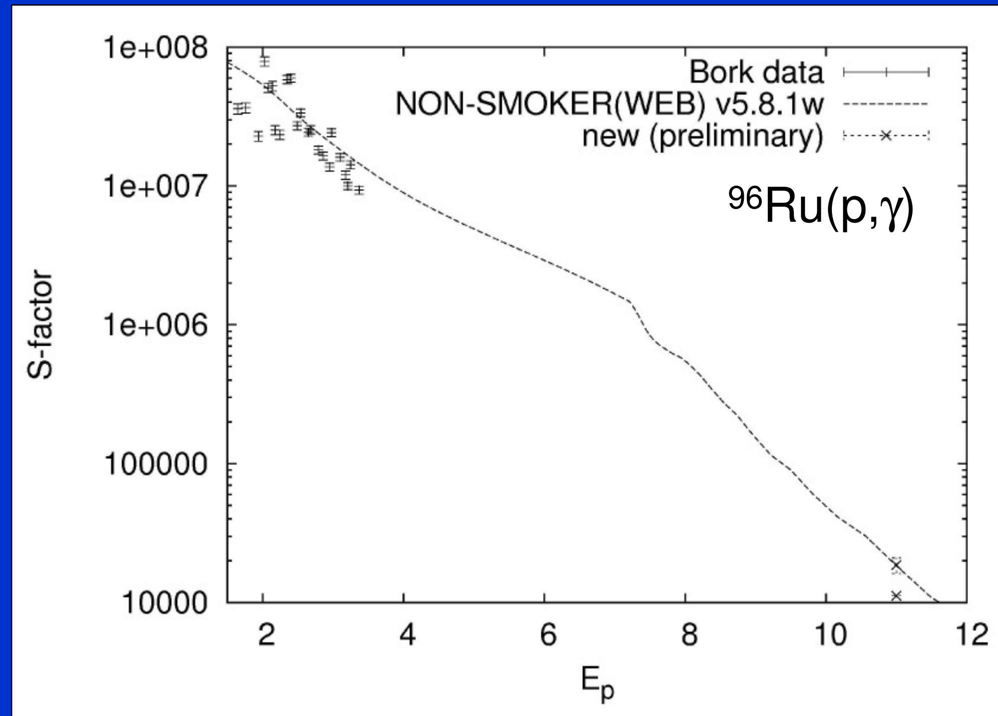
$$\sigma \propto \frac{\langle T_{\mathrm{entrance}} \rangle \langle T_{\mathrm{exit}} \rangle}{\langle T_{\mathrm{total}} \rangle}$$

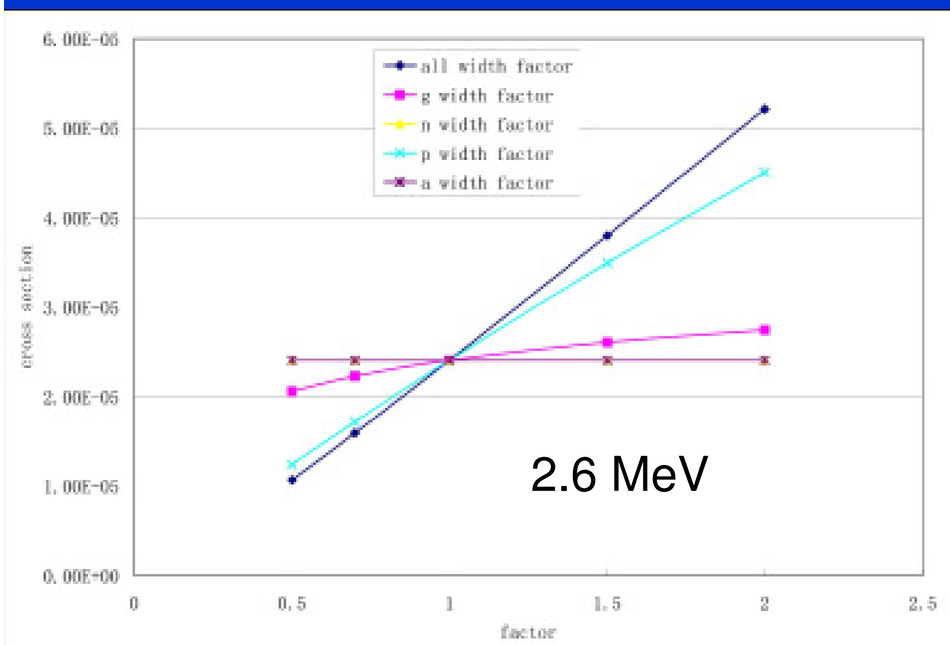


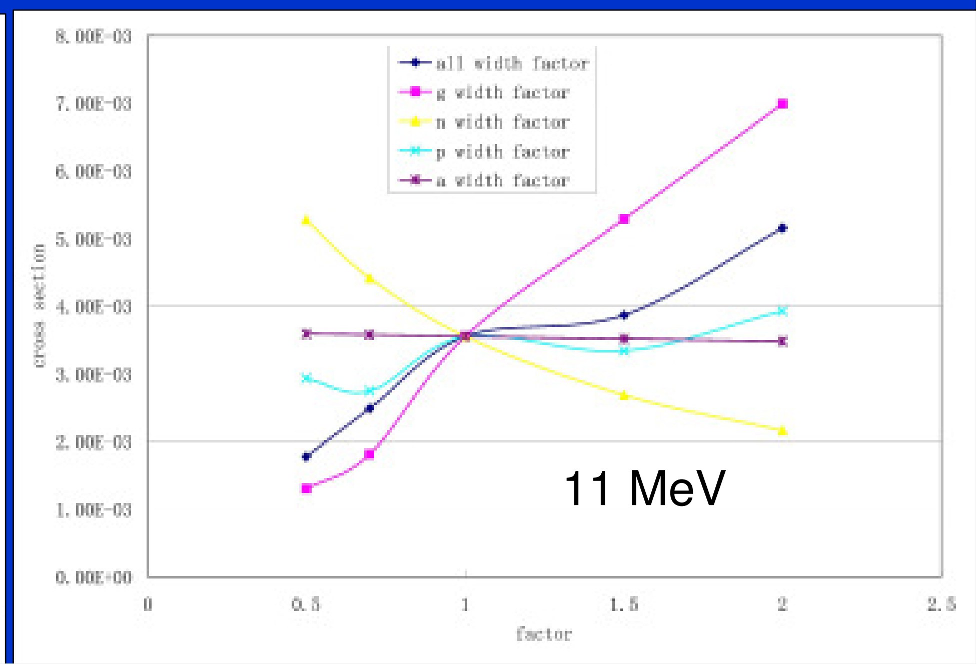


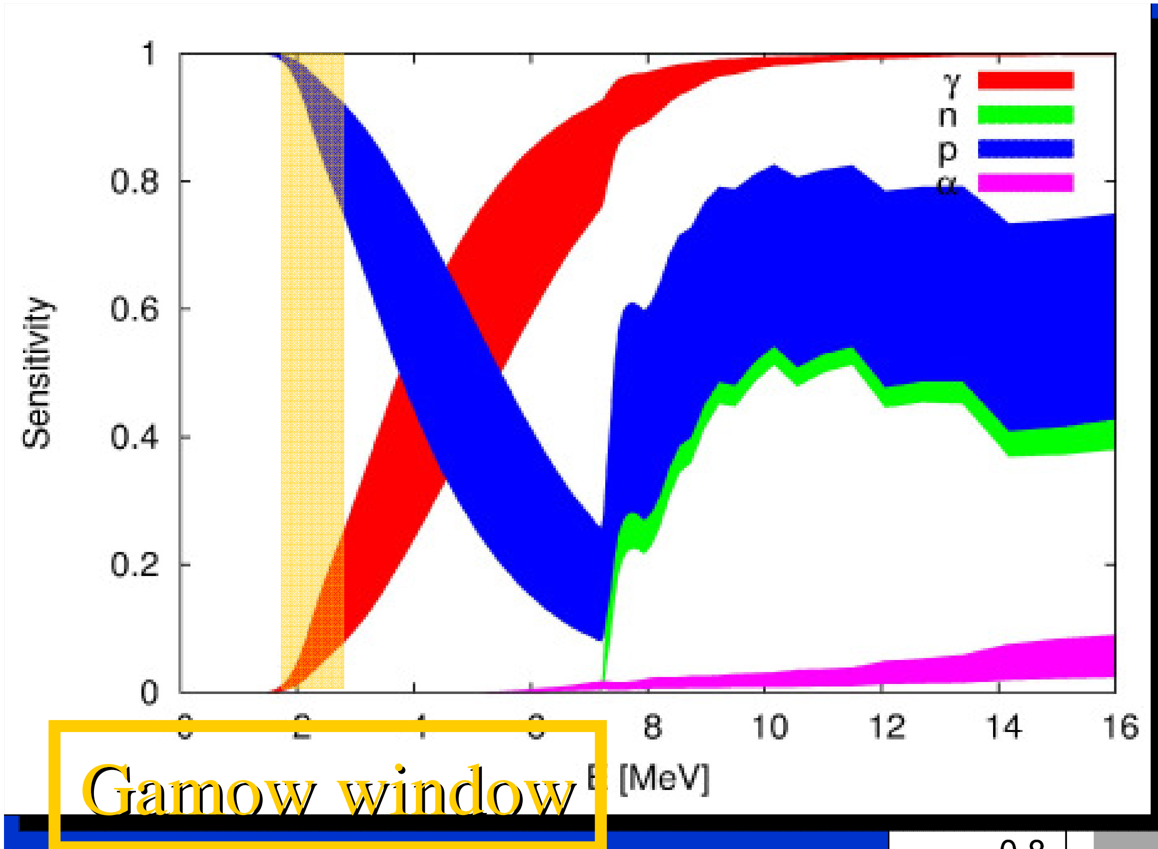
Sensitivity to Averaged Widths

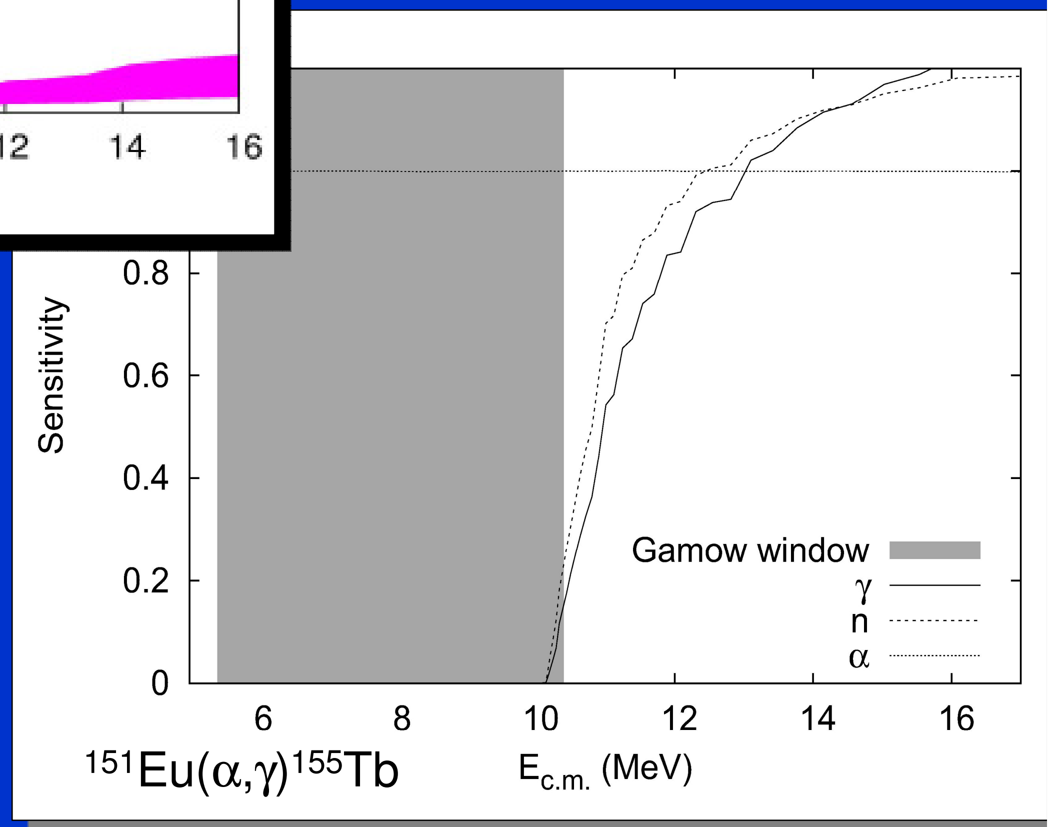
Data at higher energies do not (always) provide the information needed at astrophysical energies








1
0.8
0.6
0.4
0.2
0
Sensitivity
γ
n
p
α
8
10
12
14
16
[MeV]
Gamow window


0.8
0.6
0.4
0.2
0
Sensitivity
Gamow window
γ
n
α
6
8
10
12
14
16
$^{151}Eu(\alpha,\gamma)^{155}Tb$
$E_{c.m.}$ (MeV)

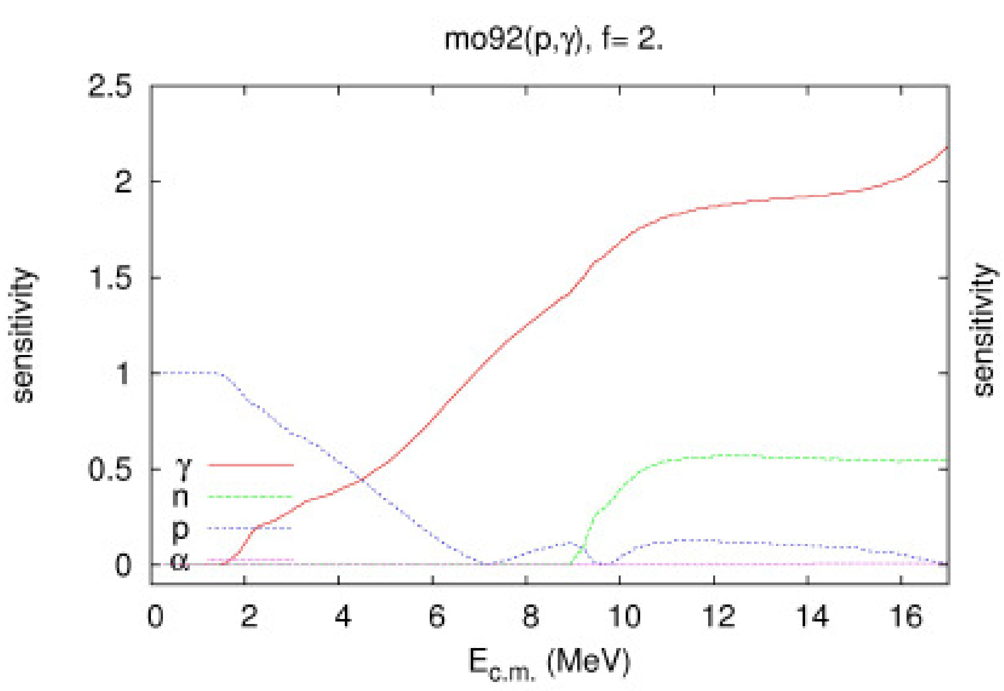
It is better to look at the rates than at the cross sections:

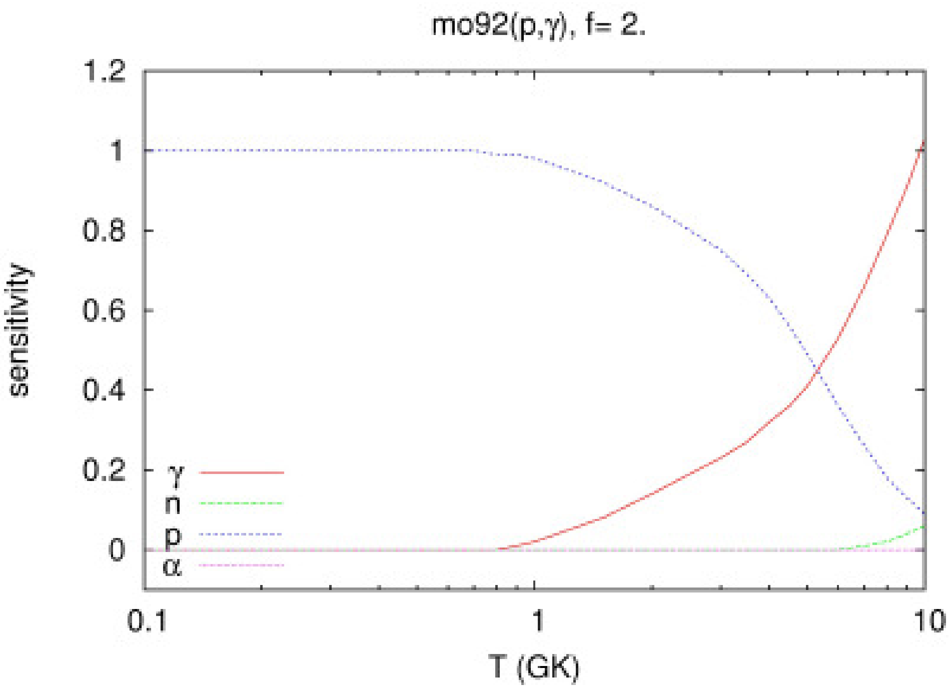
- Rates are the relevant quantities
- No need to separately compute the Gamow window

Examples relevant to the γ-process

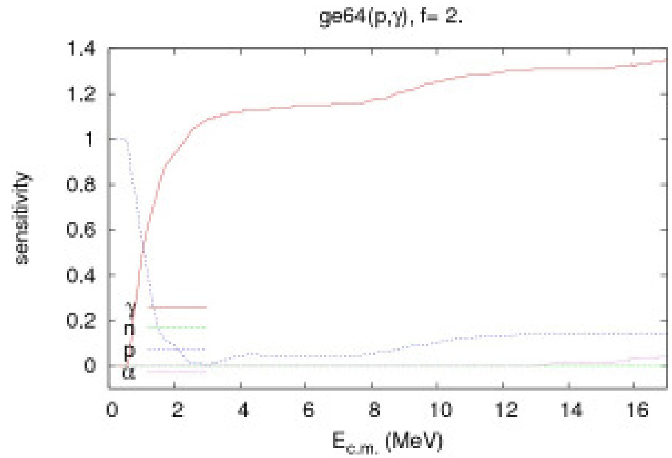
cross section sensitivity

rate sensitivity

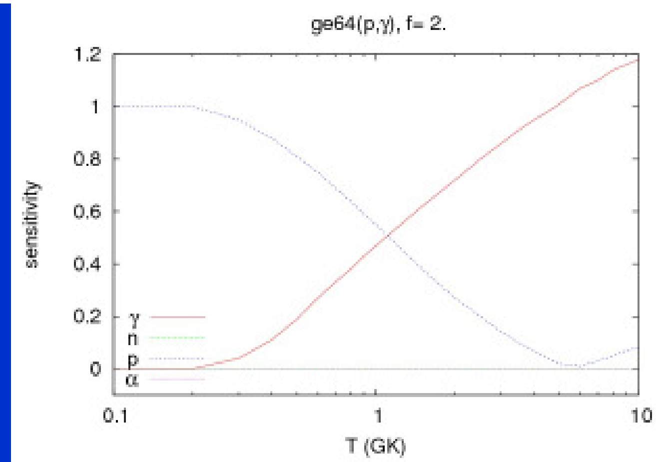


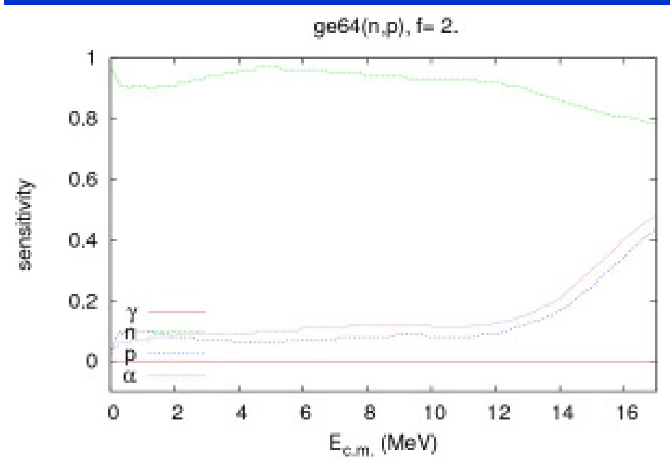


calculations performed with SMARAGD v0.8.1s

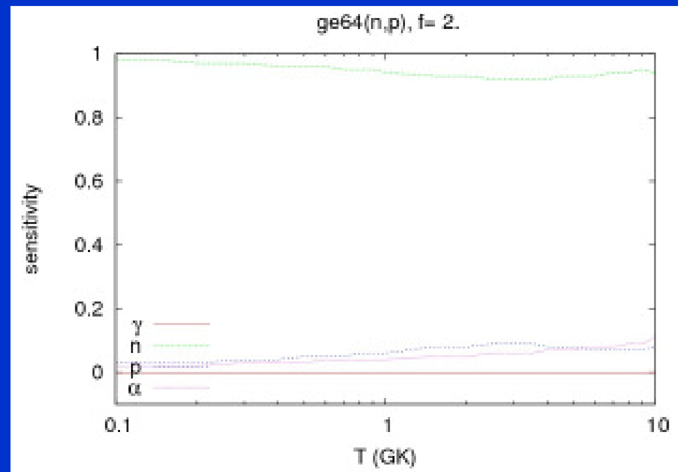


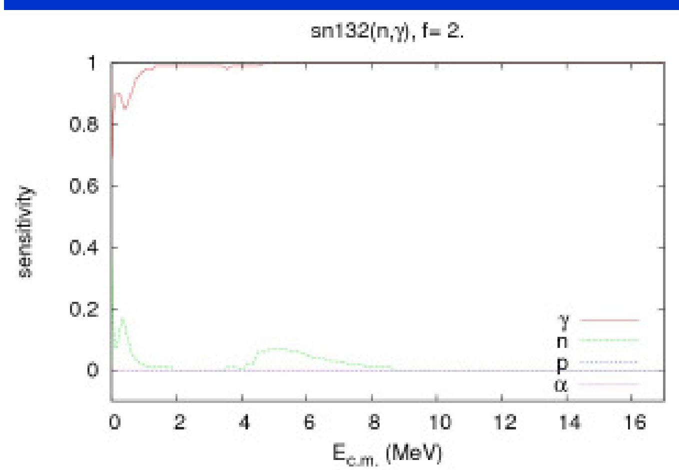
rp-process



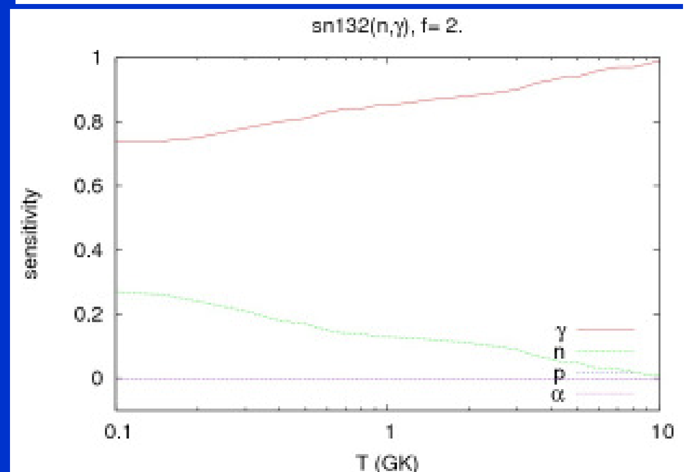


νp-process



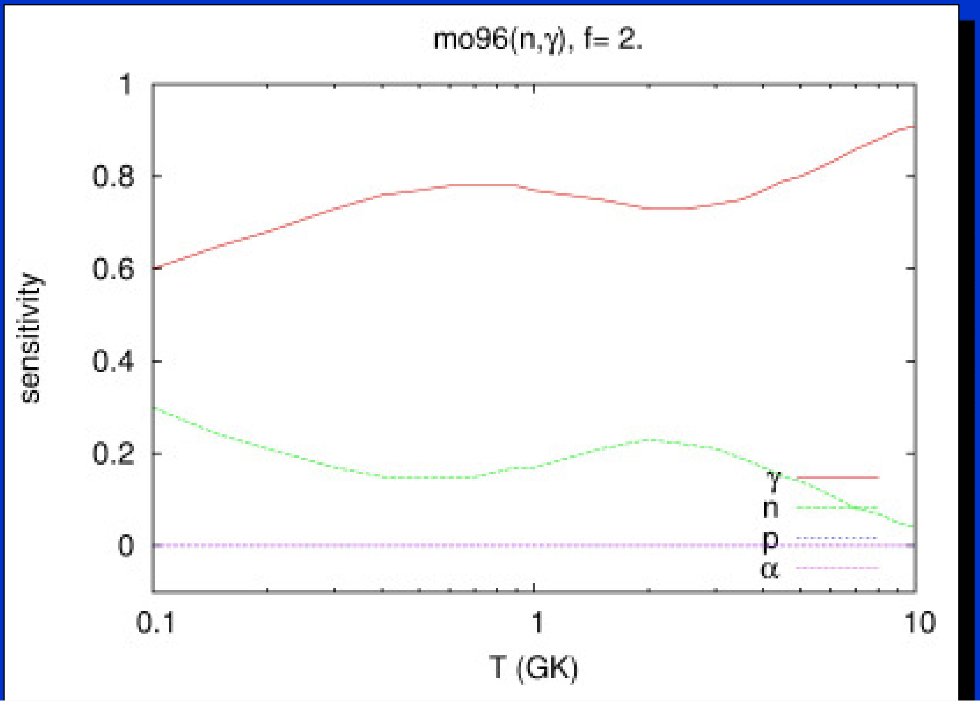


r-process



γ-Width Important?

- Not in astrophysical charged particle capture!
- But in neutron capture (and inverse) because neutron width always larger.
- However: Note the relevant γ-energies which have most impact!
  - quite similar in s-, r-, p-process



Relevant energy windows

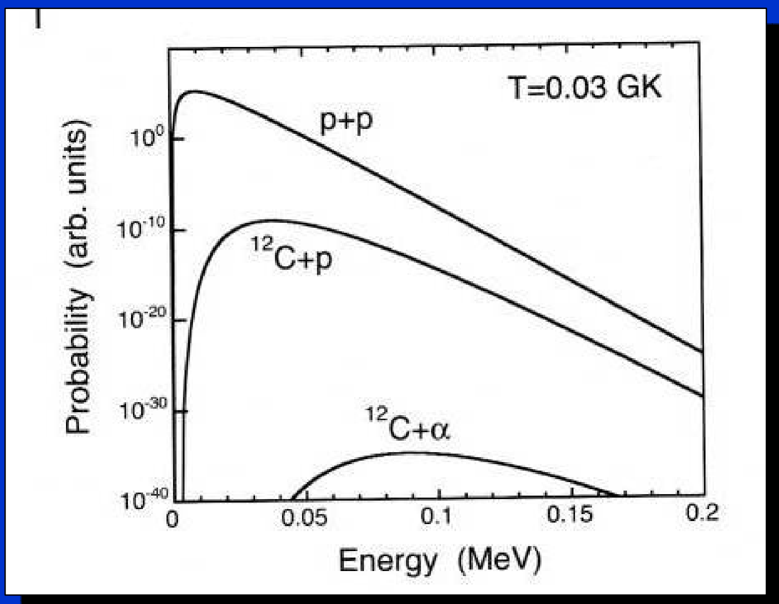
Relevant Energies – Gamow Window

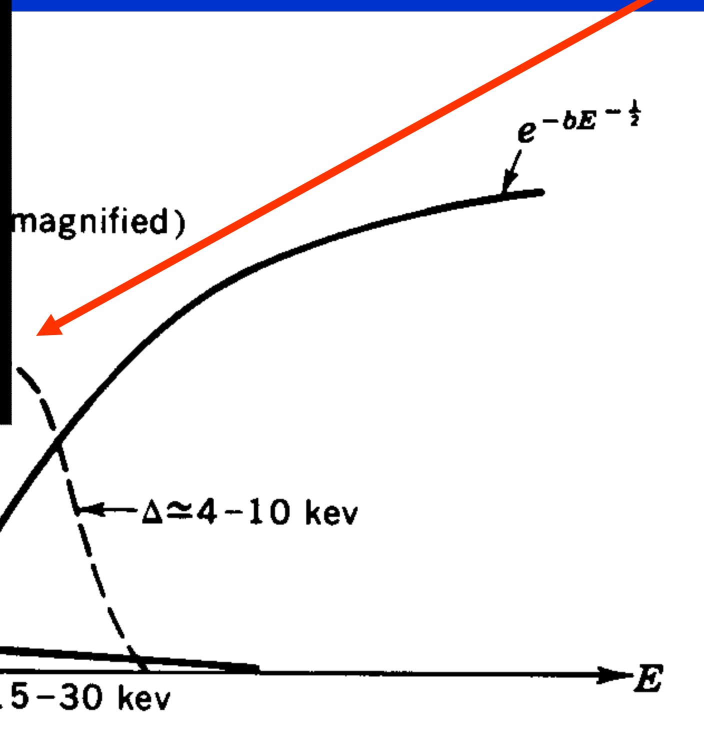
for charged particle reactions

$$<\sigma v>=\sqrt{\frac{8}{\pi\mu}}(kT)^{-3/2}\int \sigma(E)E\,\mathrm{e}^{-\frac{E}{kT}}\,dE=\sqrt{\frac{8}{\pi\mu}}(kT)^{-3/2}\int S(E)\mathrm{e}^{-\left(\frac{b}{\sqrt{E}}+\frac{E}{kT}\right)}dE$$

Gamow Peak

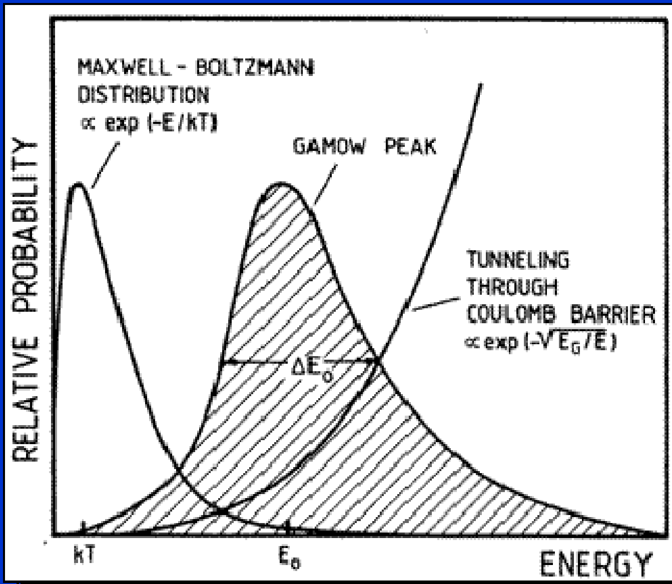
$$\sigma=\frac{1}{E}\mathrm{e}^{-b/\sqrt{E}}S(E)$$



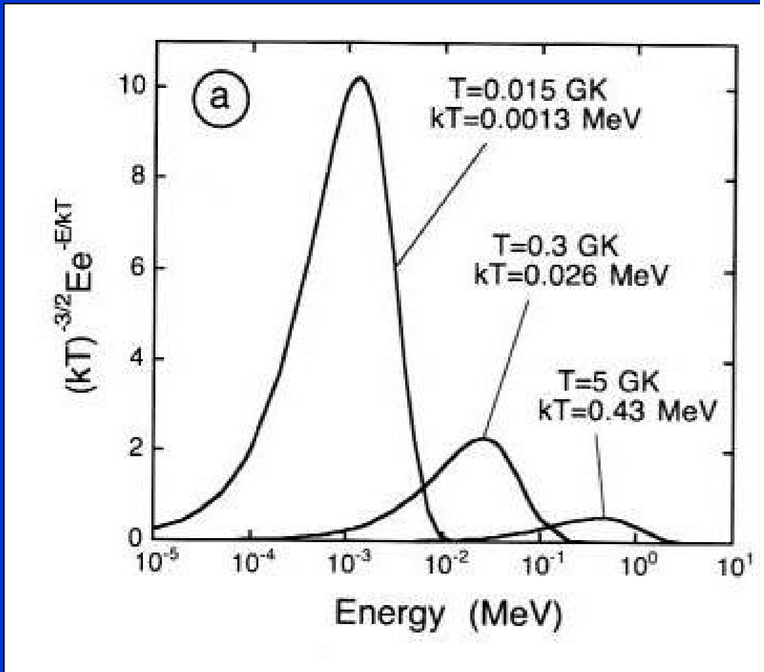


Note: relevant cross section in tail of M.B. distribution, much larger than kT (very different from n-capture !)

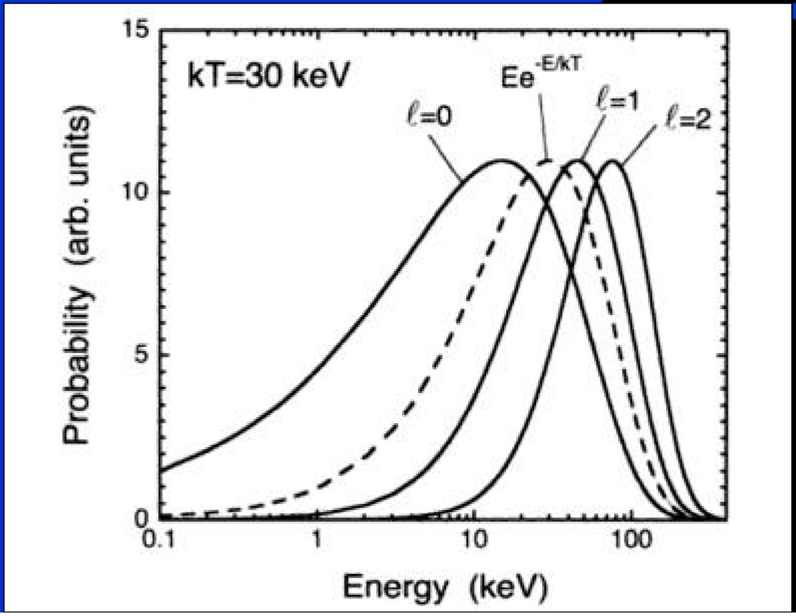
„Gamow peak“ for neutrons



Rolfs & Rodney 1985



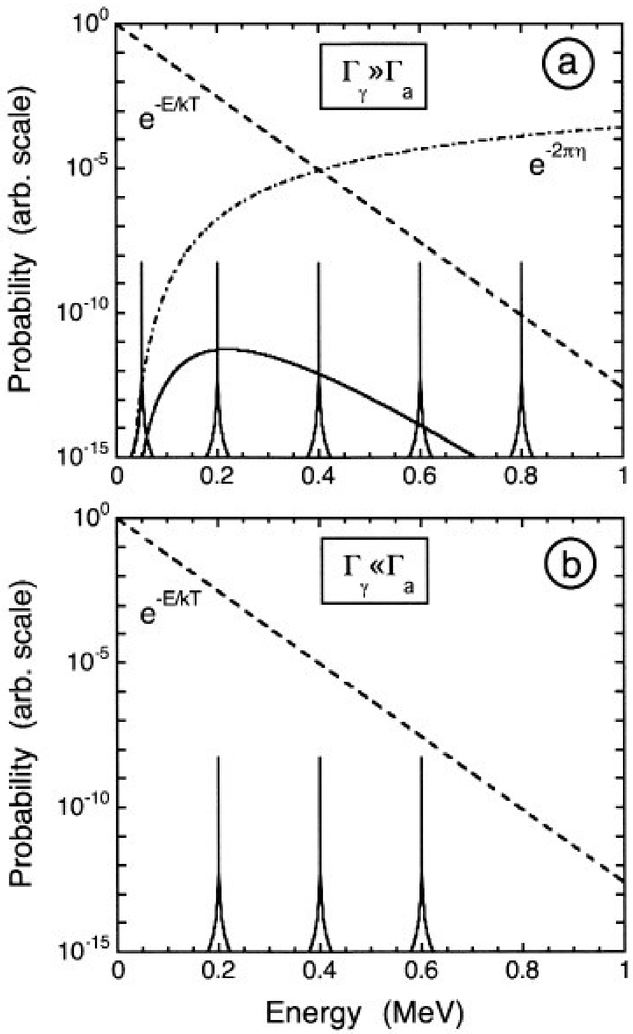
Neutrons have typical energy $kT = T_9/11.605$ MeV.



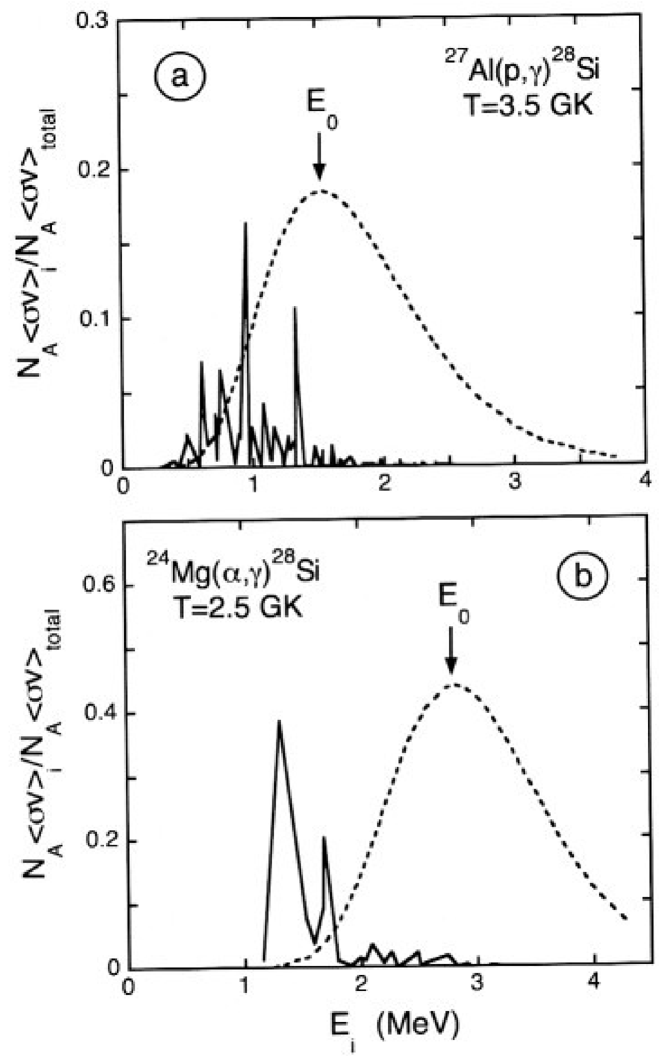
Iliadis 2006

Limitation of Gamow peak concept

Narrow resonances can also be important below the Gamow window when width of exit channel smaller than width of entrance channel!



Iliadis 2006



Newton et al 2007

Revised Gamow peaks

PHYSICAL REVIEW C **81**, 045807 (2010)

Relevant energy ranges for astrophysical reaction rates

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(Received 11 February 2010; published 29 April 2010)

Effective energy wi... (p,γ), (p,n), (p,α), (α,γ), (α,n), (α,p), (n,γ), ... n dripline are calculated using theoretical cros... for the relevant energy ranges are not valid fo... e nucleosynthesis. The influence of the energy... w windows is discussed and the results are pre...

$$\sigma \propto \frac{\langle T_{\text{entrance}} \rangle \langle T_{\text{exit}} \rangle}{\langle T_{\text{total}} \rangle}$$

The ... rtheless, is often ...

only valid when entrance channel determines energy dependence of cross section!

$$\mathcal{I}(E) \approx \mathcal{I}_{\text{max}} e^{-[4(E-E_0)^2]/\Delta^2}, \quad (7)$$

where $\mathcal{I}_{\text{max}} = \exp[-3E_0/(kT)]$ is the maximal value of the product of the two exponentials in Eq. (3) and $\Delta = 4\sqrt{E_0 kT/3}$ is the $1/e$ width of the peak. Inserting the proper numerical factors and units in Eqs. (6) and (7) leads to the more practical form [1,2,4]

$$E_0 = 0.12204\left(\mu_A Z_1^2 Z_2^2 T_9^2\right)^{\frac{1}{3}}, \quad (8)$$

$$\Delta = 0.23682\left(\mu_A Z_1^2 Z_2^2 T_9^5\right)^{\frac{1}{6}}. \quad (9)$$

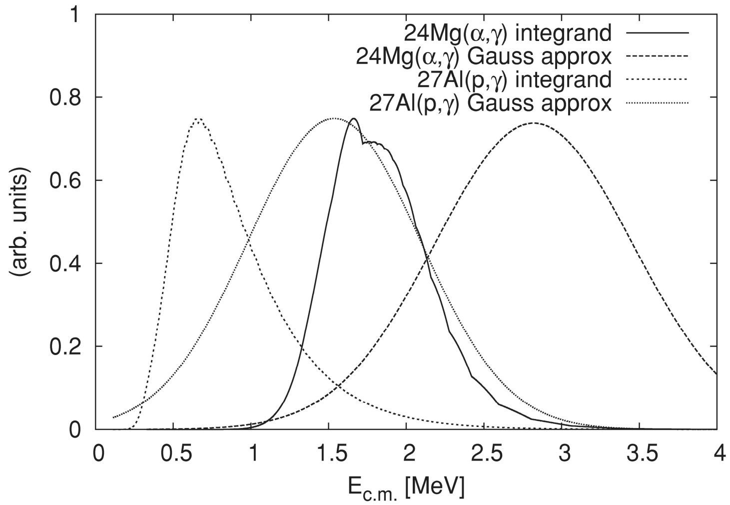


FIG. 5. Comparison of actual reaction rate integrand $\mathcal{F}$ and Gaussian approximation of the Gamow window for the reactions ^{24}Mg$(\alpha,\gamma)^{28}$Si at $T = 2.5$ GK and ^{27}Al$(p,\gamma)^{28}$Si at $T = 3.5$ GK. The integrands and Gaussians have been arbitrarily scaled to yield similar maximal values.

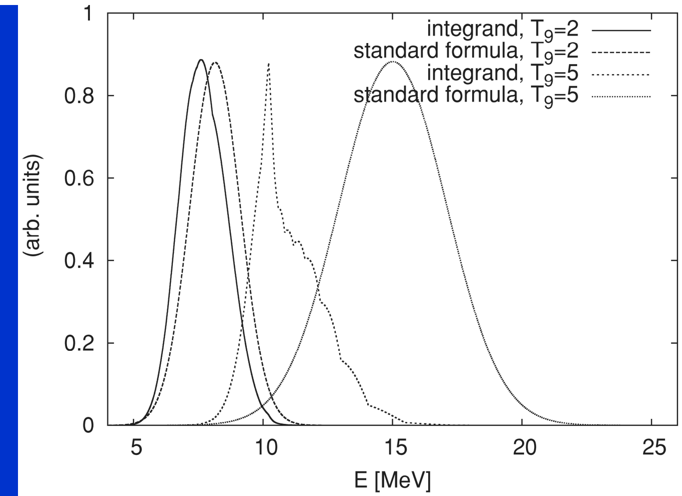


FIG. 6. Comparison of actual reaction rate integrands $\mathcal{F}$ and Gaussian approximations of the Gamow window for the reaction ^{169}Tm$(\alpha,\gamma)^{173}$Lu at $T = 2$ and 5 GK. The integrands and Gaussians have been arbitrarily scaled to yield similar maximal values. While the shift is small for $T_9 = 2$, it is about 5 MeV at $T_9 = 5$. Also, the [illegible] the integrand can be clearly seen at $T_9 = 5$.

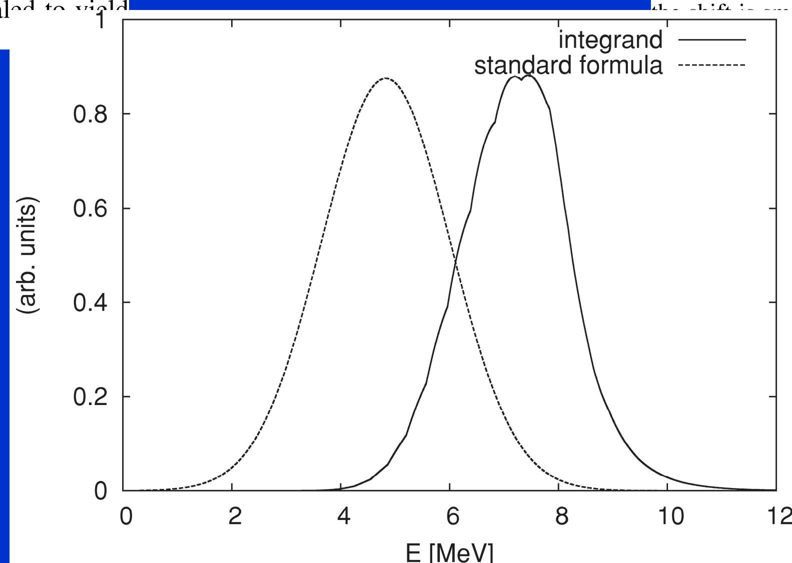


FIG. 9. Comparison of the actual reaction rate integrand $\mathcal{F}$ and the Gaussian approximation of the Gamow window for the reaction ^{112}Sn$(p,\alpha)^{109}$In at $T = 5$ GK. The two curves have been arbitrarily scaled to yield similar maximal values. The maximum of the integrand is shifted by several mega–electron volts to energies higher than the maximum E_0 of the Gaussian.

TABLE I. Effective energy windows $\widetilde{E}_{\rm hi} - \widetilde{\Delta} \leqslant E \leqslant \widetilde{E}_{\rm hi}$ for a given plasma temperature T. Also listed is the energy $\widetilde{E}_0$ of the maximum in the reaction rate integrand and its shift δ relative to the standard formula. The latter is $\delta = \widetilde{E}_0 - E_0$ relative to the location of the Gamow peak E_0 for charged-particle-induced reactions and $\delta = \widetilde{E}_0 - E_{\rm MB}$ relative to the maximum of the MB distribution at $E_{\rm MB}$ for neutron-induced reactions. This table lists only a few examples. The full table is available from Ref. [7].

Target	Reaction	T (GK)	$\widetilde{E}_{\rm hi}$ (MeV)	$\widetilde{\Delta}$ (MeV)	$\widetilde{E}_0$ (MeV)	δ (MeV)
^{24}Mg	(α,γ)	2.5	2.36	1.05	1.66	−1.16
^{27}Al	(p,γ)	3.5	1.47	1.12	0.65	−0.89
^{40}Ca	(α,γ)	2.0	3.62	1.39	2.85	−0.63
		4.0	4.66	1.97	3.56	−1.97
^{60}Fe	(n,γ)	5.0	1.20	1.20	0.13	−0.30
^{62}Ni	(n,γ)	3.5	1.00	1.00	0.15	−0.15
^{106}Cd	(α,γ)	3.5	10.07	3.44	8.08	−1.17
^{120}Sn	(n,α)	5.0	9.54	4.16	6.92	+6.49
^{144}Sm	(α,γ)	3.5	11.97	3.99	9.90	−1.10
^{169}Tm	(α,γ)	2.0	9.20	2.94	7.61	−0.54
		5.0	13.20	4.27	10.22	−4.79

$$\widetilde{E}_{\rm hi} - \widetilde{\Delta} \leqslant E \leqslant \widetilde{E}_{\rm hi}.$$

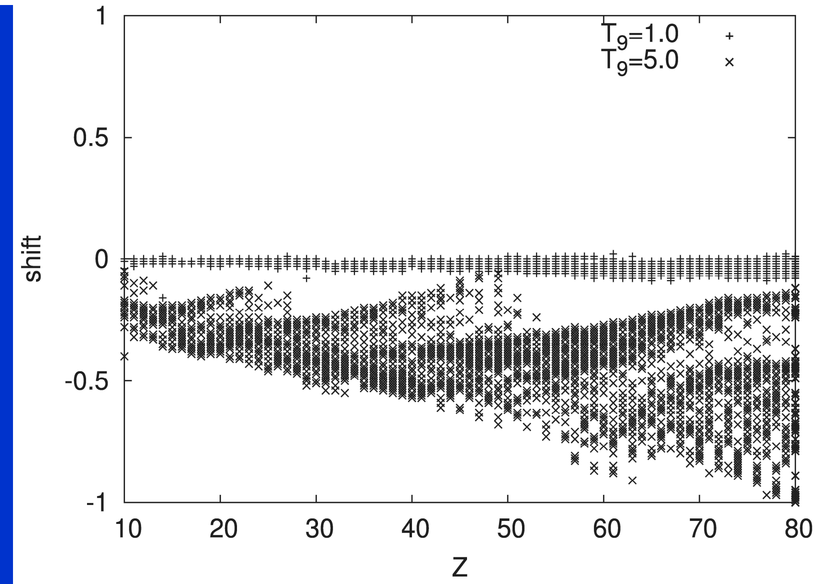


FIG. 1. Shifts δ (MeV) of the maximum of the integrand relative to E_0 of the Gaussian approximation as a function of the target charge Z for (p,n) reactions at two temperatures. Almost no shift is observed a

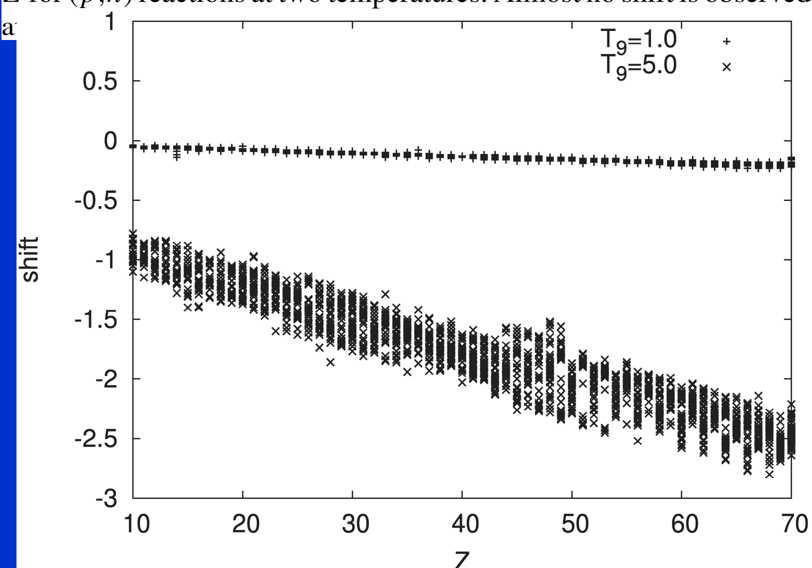


FIG. 2. Shifts δ (MeV) of the maximum of the integrand relative to E_0 of the Gaussian approximation as a function of the target charge Z for (α, n) reactions at two temperatures. Almost no shift is observed at $T_9 = 1.0$ and shifts reach a few mega–electron volts for $T_9 = 5.0$.

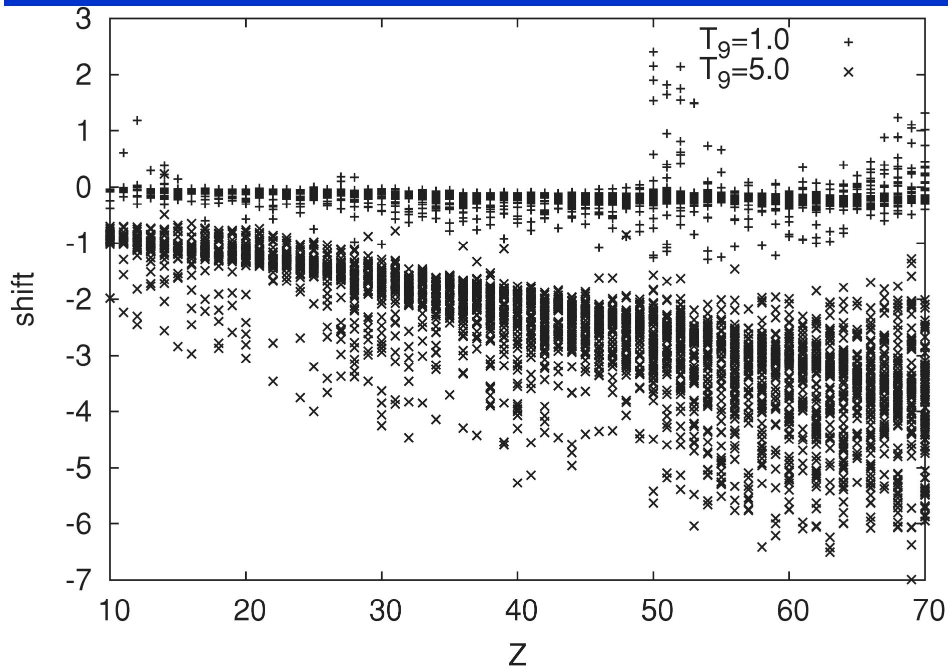


FIG. 3. Shifts δ (MeV) of the maximum of the integrand relative to E_0 of the Gaussian approximation as a function of the target charge Z for (α,γ) reactions at two temperatures. Almost no shift is observed at $T_9 = 1.0$ but shifts become large at $T_9 = 5.0$.

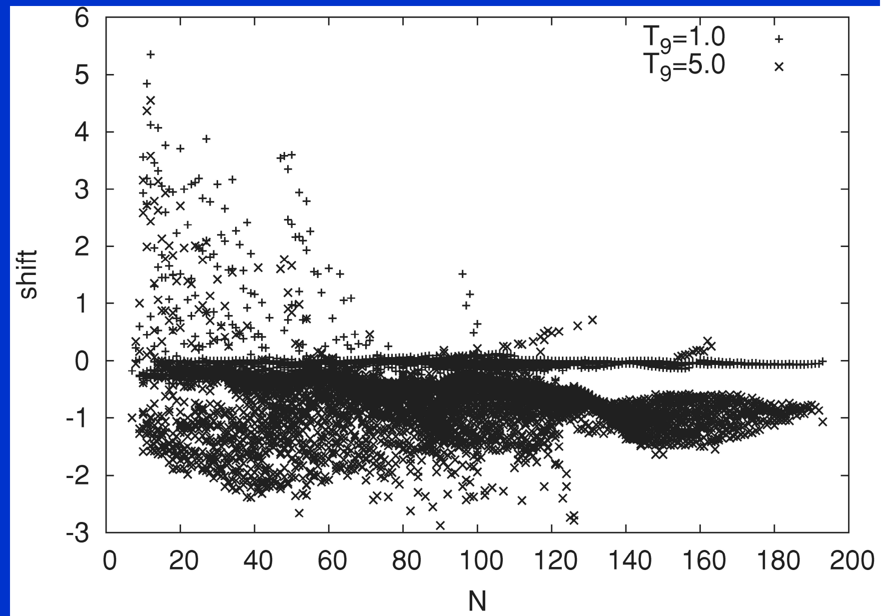


FIG. 4. Shifts δ (MeV) of the maximum of the integrand relative to E_0 of the Gaussian approximation as a function of the target neutron number N for (p,γ) reactions at two temperatures. Almost no shift is observed at $T_9 = 1.0$, except for proton-rich nuclei with a negative reaction Q value. Shifts remain smaller than for (α,γ) at $T_9 = 5.0$.

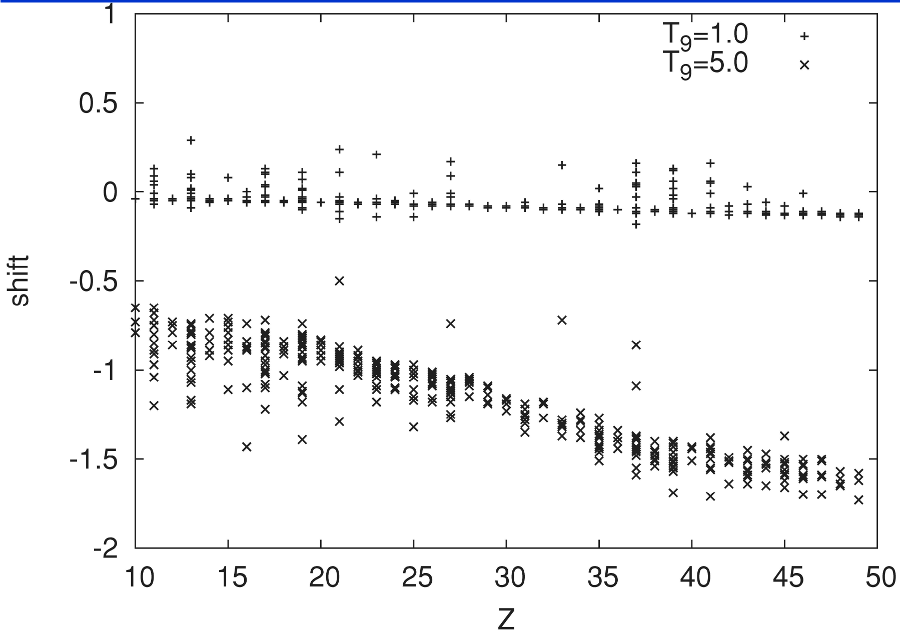


FIG. 7. Shifts δ (MeV) of the maximum of the integrand relative to E_0 of the Gaussian approximation as a function of the target charge Z for (α, p) reactions at two temperatures. Almost no shift is observed at $T_9 = 1.0$ and shifts reach a few mega–electron volts for $T_9 = 5.0$.

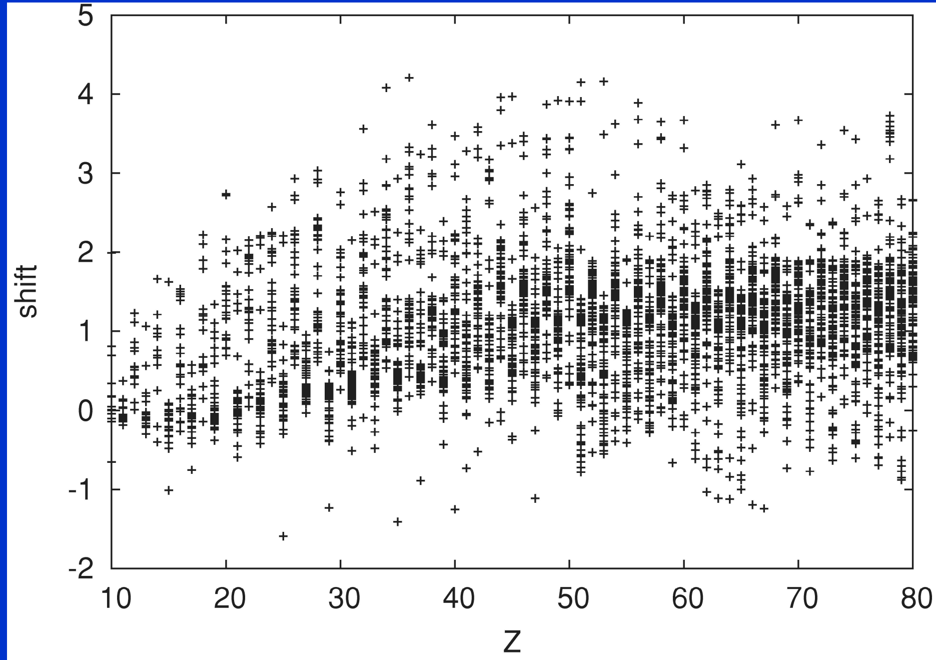


FIG. 8. Shifts δ (MeV) of the maximum of the integrand relative to E_0 of the Gaussian approximation as a function of the target charge Z for (p,α) reactions at $T = 5$ GK. The shifts are larger as for (α, p) reactions and they are positive.

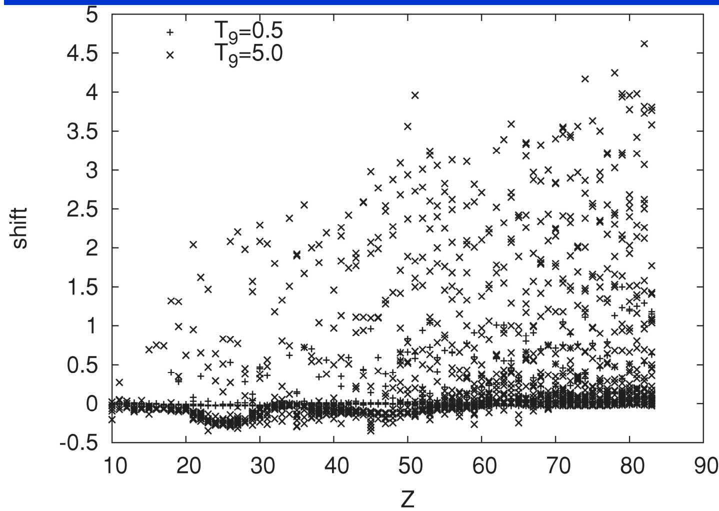


FIG. 10. Shifts δ (MeV) of the maximum of the integrand relative to $E_{\rm MB}$ as a function of the target charge Z for (n,p) reactions at two temperatures. Almost no shift is observed at $T_9 = 0.5$ but shifts become large at $T_9 = 5.0$.

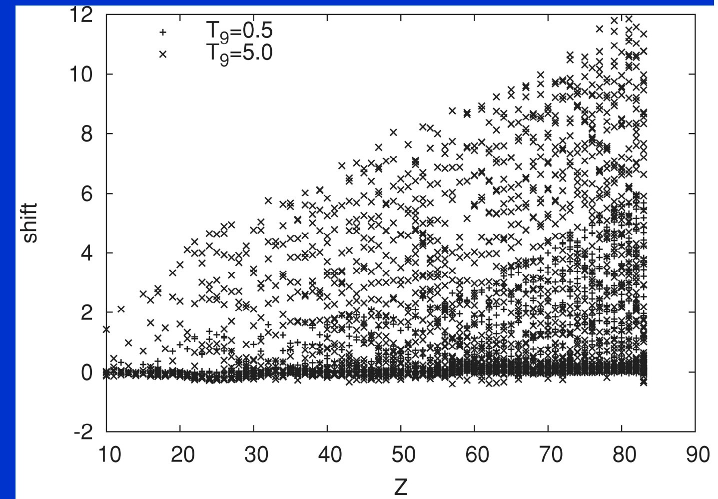


FIG. 11. Shifts δ (MeV) of the maximum of the integrand relative to $E_{\rm MB}$ as a function of the target charge Z for (n,p) reactions at
o shift is observed at $T_9 = 0.5$ but shifts
.0.

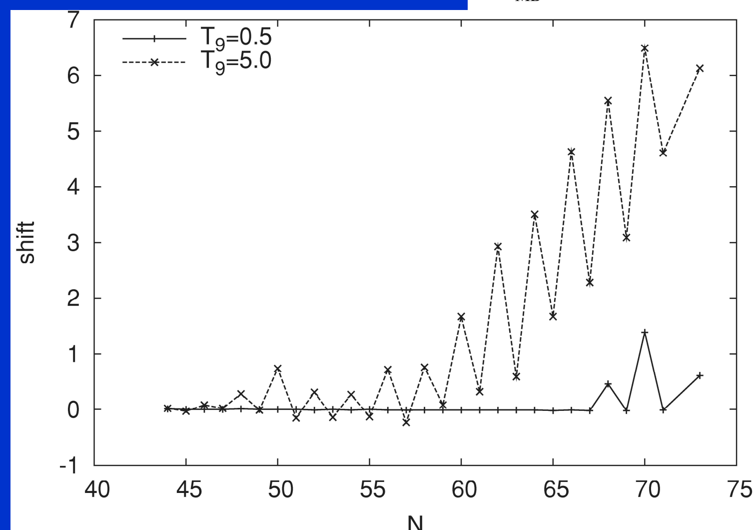


FIG. 12. Shifts δ (MeV) of the maximum of the integrand relative to $E_{\rm MB}$ as a function of the target neutron number N for (n,α) reactions on Sn isotopes at two temperatures. Almost no shift is observed at $T_9 = 0.5$ but shifts become large at $T_9 = 5.0$ for the neutron-rich isotopes with a small reaction Q value.

only valid for (n,γ):

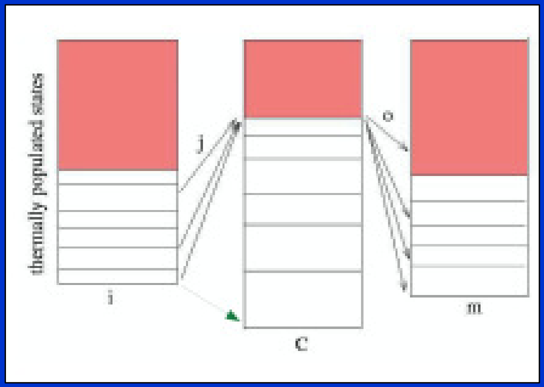
$$E_{\rm eff} \approx 0.172 T_9 \left(\ell + \frac{1}{2}\right),$$

$$\Delta_{\rm eff} \approx 0.194 T_9 \sqrt{\ell + \frac{1}{2}},$$

Ground state contribution to stellar rates

Thermally excited target nuclei

Ratio of nuclei in a thermally populated excited state to nuclei in the ground state is given by the Saha Equation:



$$\frac{n_{\mathrm{ex}}}{n_{\mathrm{gs}}} = \frac{g_{\mathrm{ex}}}{g_{\mathrm{gs}}} e^{-\frac{E_x}{kT}}$$

$$g = (2J+1)$$

Ratios of order 1 for $E_x \sim kT$

For nuclear astrophysics, location of Gamow window has to be compared to average level spacing in nuclei.

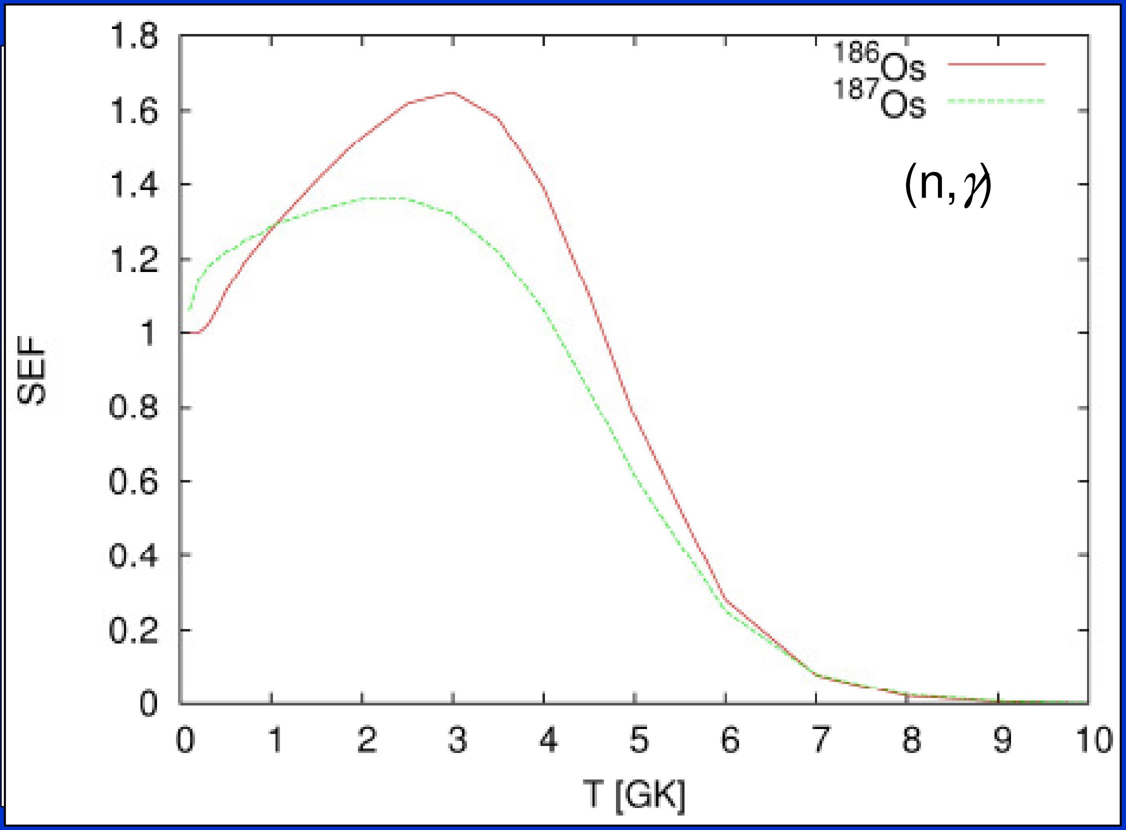
- Only small correction for:
    - light nuclei (level spacing several MeV)
    - Gamow window at low energy: at low T
- **LARGE correction**, when
    - low lying (~100 keV) excited state(s) exist(s) in the target nucleus
    - temperatures are high (explosive nucleosynthesis)
    - the populated state has a very different rate

The correction for this effect has to be calculated.

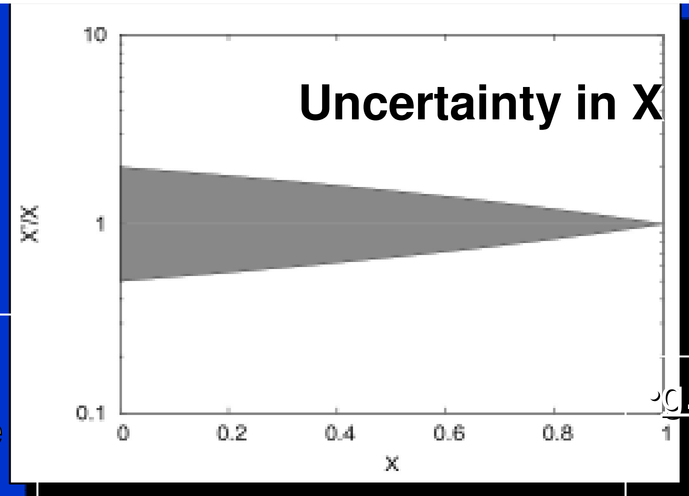
Thermally excited target nuclei

Example for the impact of temperature on the Stellar Enhancement Factor (SEF).

$$f_{\text{SEF}} = \frac{R^*}{R_0} = \frac{\langle \sigma v \rangle^*}{\langle \sigma v \rangle^{\text{lab}}}$$



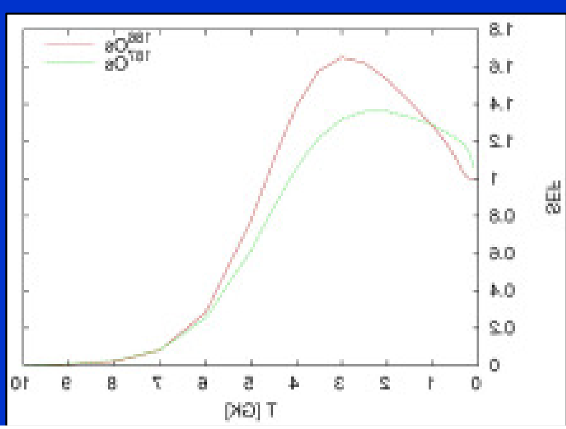
$$f_{\text{SEF}} = \frac{R^*}{R_0}$$



$$X = \frac{R_0}{R^* G_0}$$

• Stellar Enhancement (SEF)

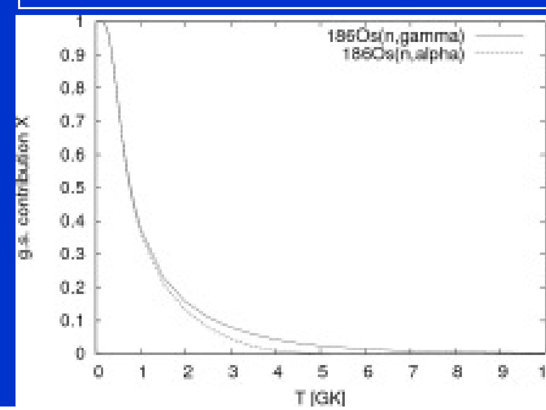
- compares rate to g.s. rate
- =1 at T=0
- may assume any value at low T
- asymptotically goes to $1/G_0$, i.e. to 0.
- Uncertainty scales with G_0 but is not related to value



• g.s. contribution (X)

- gives g.s. contribution to stellar rate
- =1 at T=0
- confined to 0<=X<=1
- monotonically decreasing to 0
- Uncertainty scales with G_0 and is related to X:
  - $u=(1-X)u'$

$$X = \frac{1}{G_0 f_{\text{SEF}}}$$



“The s-process is the best understood nucleosynthesis process” ?

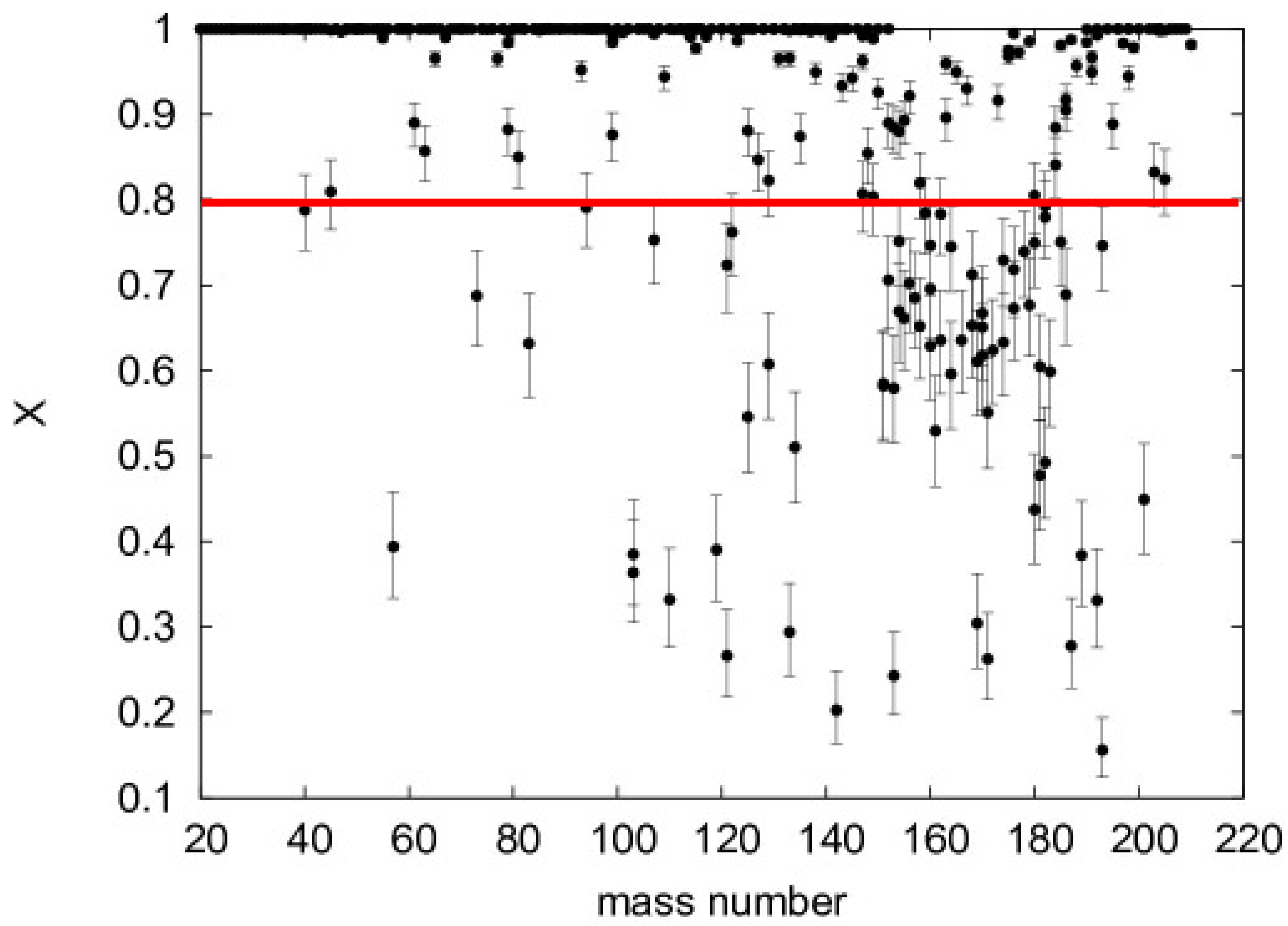
“The s-process is the best experimentally constrained nucleosynthesis process” ?

This is based on two facts:

1. High-precision neutron capture data available (error <1-2%)
2. Stellar enhancement factors close to unity (or only on the order of 1.2 or so)

<u>But 2) is pure theory with complicated uncertainty and SEF is not the relevant quantity!</u>

How well can experiments constrain s-process neutron capture?



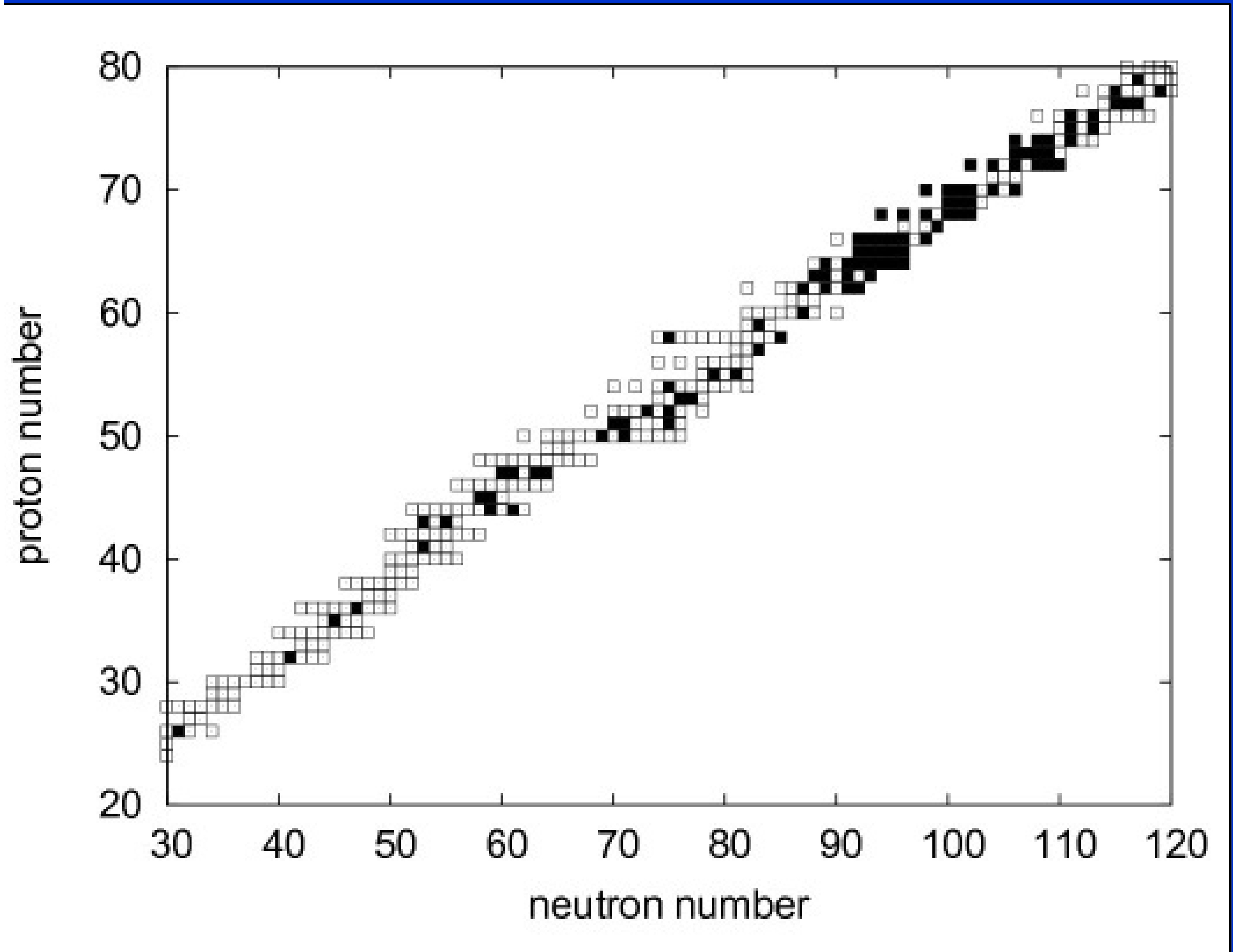
X directly also gives the maximally possible reduction in (theory) uncertainty by experiments!

- Nuclides from KADoNiS
- (n,γ) at kT=30 keV

Rauscher P. Mohr, I. Dillmann, R. Plag; Ap. J. 738 (2011) 143.

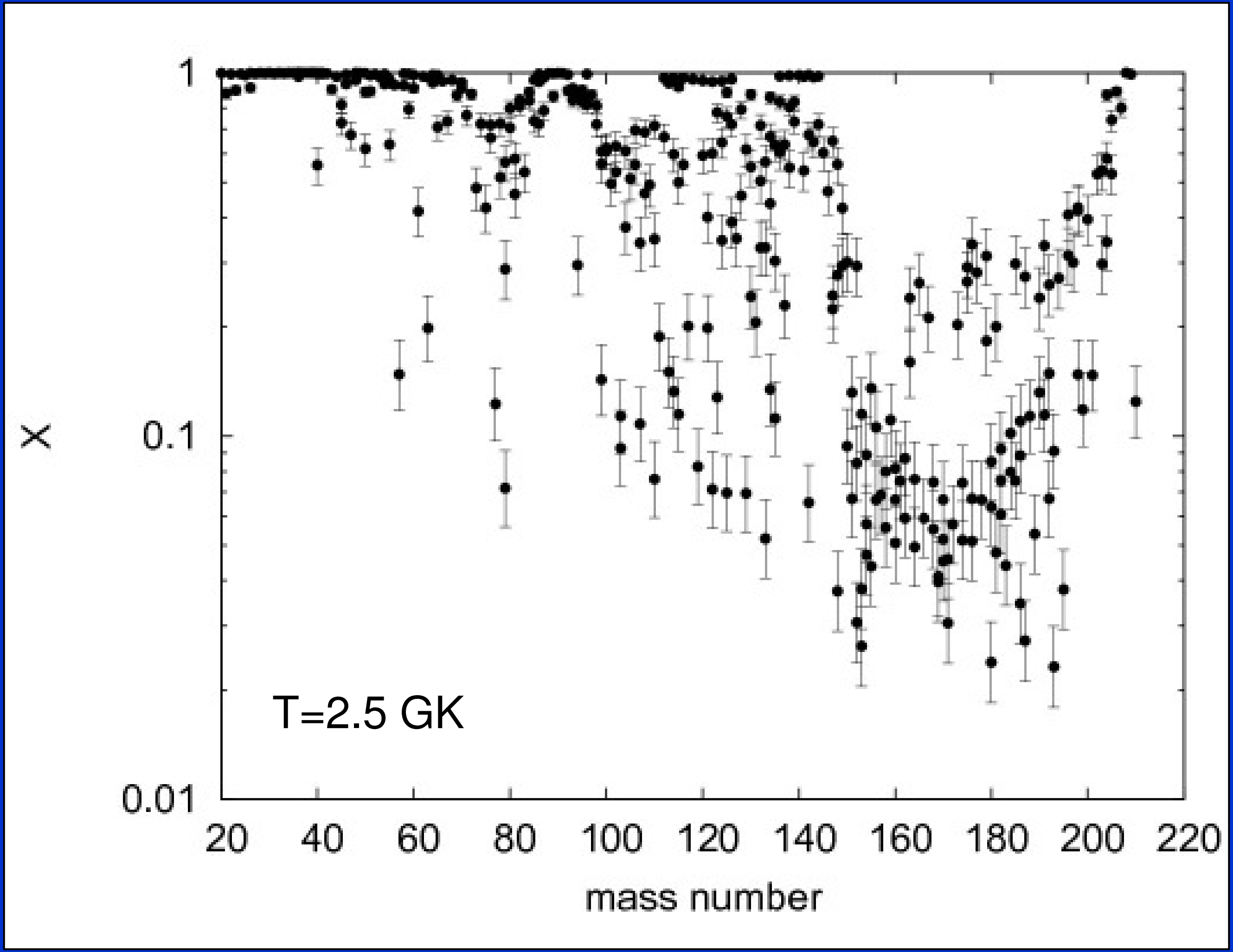
G_0 known for s-process conditions!

How well can experiments constrain s-process neutron capture?



Black squares are nuclei for which error cannot be reduced by more than 80%

How well can experiments constrain γ-process neutron capture?



Effective cross section, reciprocity, and the Q-value rule

Reaction Rate (MB)

$$r_{12} = \frac{1}{1+\delta_{12}} n_1 n_2 \langle \sigma^* v \rangle_{12} = \frac{1}{1+\delta_{12}} \rho^2 Y_1 Y_2 N_A^2 \langle \sigma^* v \rangle_{12}$$

Number of reactions per time and volume

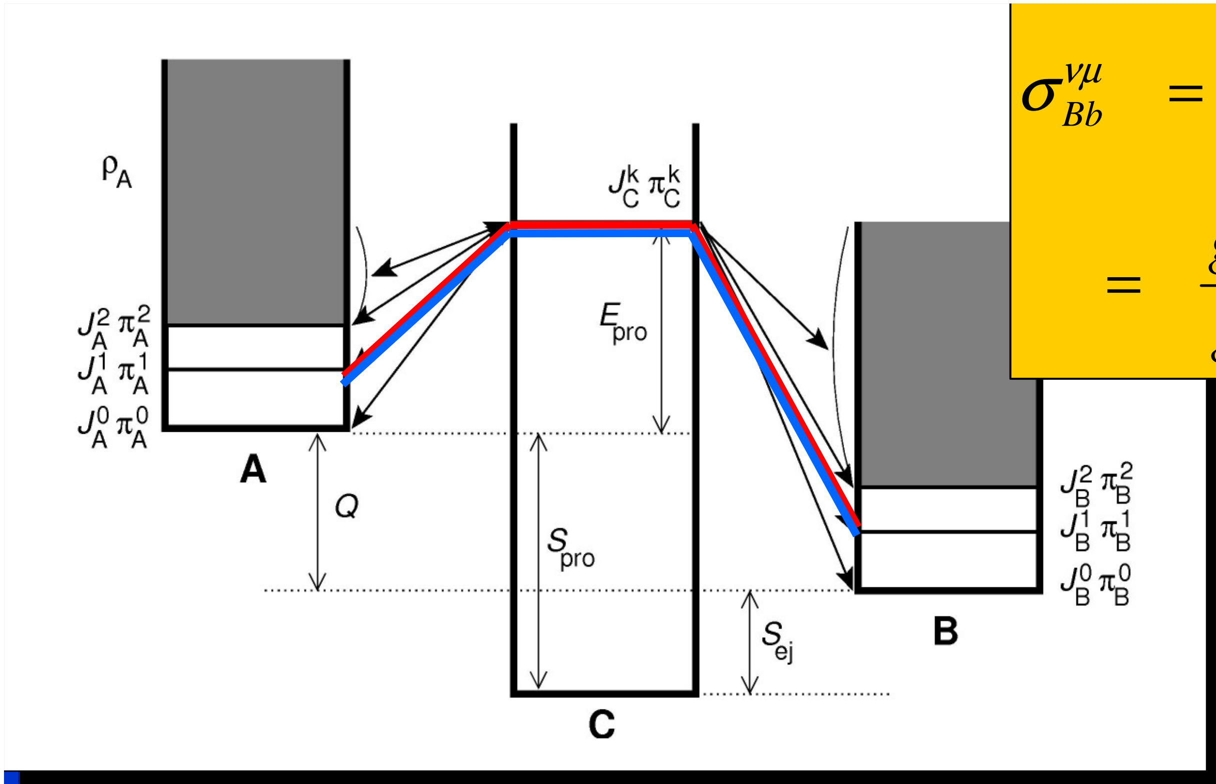
$$\langle \sigma^* v \rangle \propto \int \sigma^* E e^{-E/(kT)} dE$$

Stellar reactivity

Incorrect!

$$\sigma^* = \frac{\sum_i (2J_i + 1) \sigma_i e^{-E_i/kT}}{\sum_i (2J_i + 1) e^{-E_i/kT}}$$

Stellar cross section



$$\sigma_{Bb}^{\nu\mu} = \frac{(2J_A^\mu+1)(2J_a+1)}{(2J_B^\nu+1)(2J_b+1)}\frac{k_{Aa}^2}{k_{Bb}^2}\sigma_{Aa}^{\mu\nu} = \frac{g_A^\mu g_a}{g_B^\nu g_b}\frac{m_A}{m_B}\frac{E_A}{E_B}\sigma_{Aa}^{\mu\nu}$$

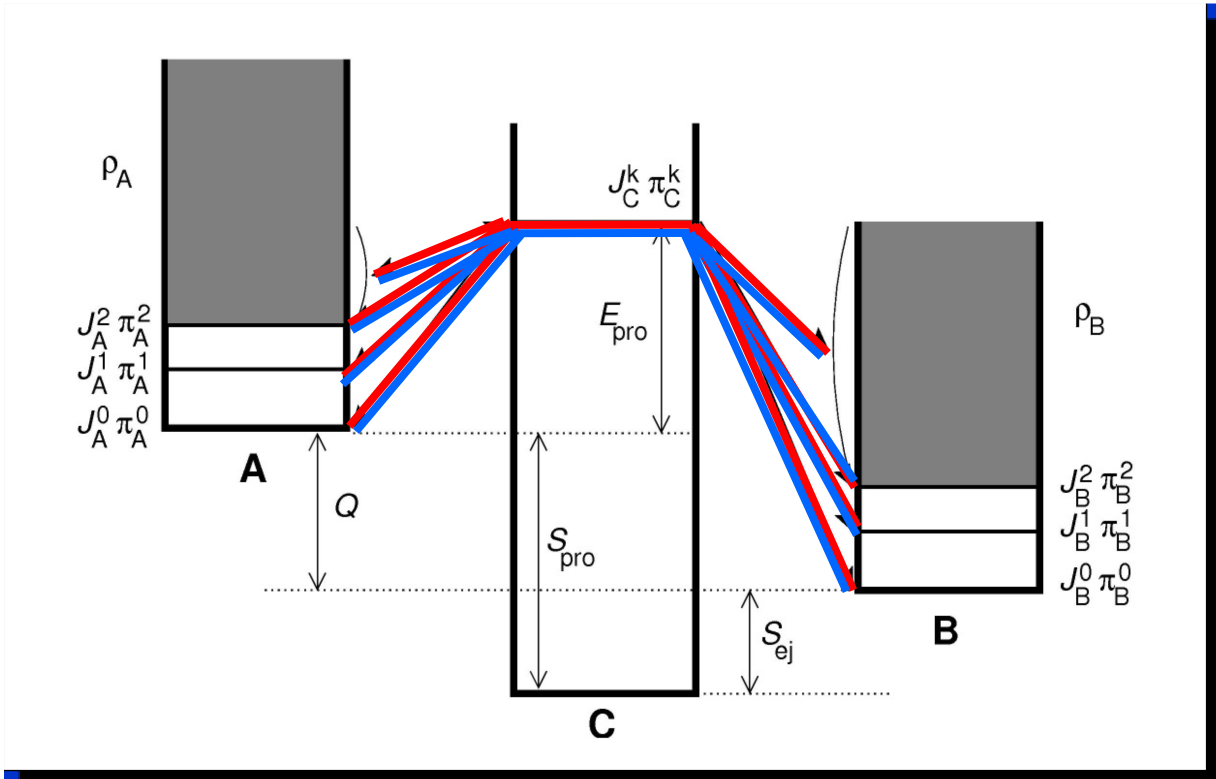
Reciprocity relation

$$\sigma_{\text{lab}} = \sigma_{Aa}^0 = \sum_\nu \sigma_{Aa}^{0\nu}$$

Lab cross section; no reciprocity with σ_{Bb}^0

$$\left(\text{in general}: \sigma_{Aa}^\mu = \sum_\nu \sigma_{Aa}^{\mu\nu}\right)$$

Fowler '74



$$\sigma_{Bb}^{\nu\mu} = \frac{g_A^{\mu} g_a}{g_B^{\nu} g_b} \frac{m_A}{m_B} \frac{E_A^{\mu}}{E_B^{\nu}} \sigma_{Aa}^{\mu\nu}$$

Reciprocity relation

For fun, let's postulate "effective" cross section:

$$\sigma_{Aa}^{\text{eff}} = \sum_{\mu}\sum_{\nu} \frac{g_A^{\mu}}{g_A^{0}} \frac{E_A^{\mu}}{E_A^{0}} \sigma_{Aa}^{\mu\nu} = \sum_{\mu} \frac{g_A^{\mu}}{g_A^{0}} \frac{E_A^{\mu}}{E_A^{0}} \sigma_{Aa}^{\mu}$$

$$\sigma_{Bb}^{\text{eff}} = \sum_{\nu} \frac{g_B^{\nu}}{g_B^{0}} \frac{E_B^{\nu}}{E_B^{0}} \sigma_{Bb}^{\nu}$$

Reciprocity again!!

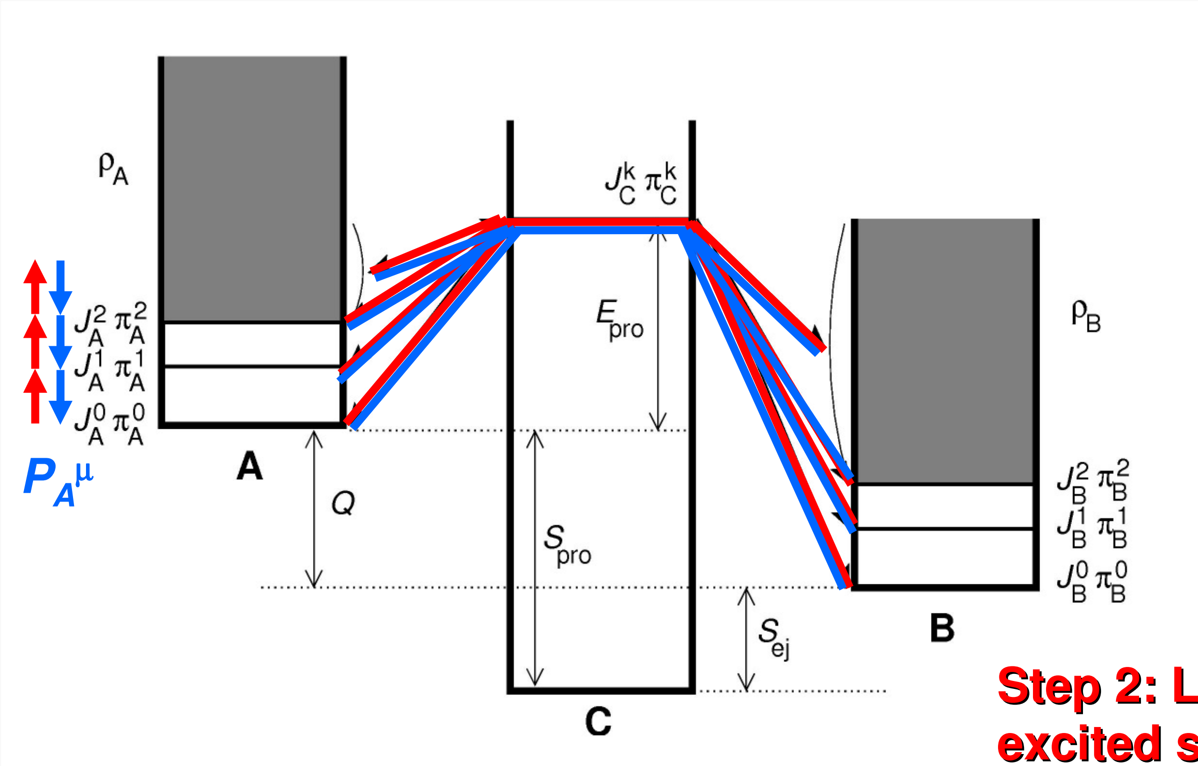
$$\sigma_{Bb}^{\text{eff}} = \frac{g_A^{0} g_a}{g_B^{0} g_b} \frac{m_A}{m_B} \frac{E_A^{0}}{E_B^{0}} \sigma_{Aa}^{\text{eff}}$$

or

$$\boxed{\sigma_{Bb}^{\text{eff}} E_B^{0}} = \frac{g_A^{0} g_a}{g_B^{0} g_b} \frac{m_A}{m_B} \boxed{E_A^{0} \sigma_{Aa}^{\text{eff}}}$$

But: unmeasureable!

Fowler '74



$$\sigma_{Aa}^{\rm eff} = \sum_\mu \frac{g_A^\mu}{g_A^0}\frac{E_A^\mu}{E_A^0}\sigma_{Aa}^\mu$$

Effective c.s.

$$\sigma_{Bb}^{\rm eff} E_B^0 = \frac{g_A^0 g_a}{g_B^0 g_b}\frac{m_A}{m_B} E_A^0 \sigma_{Aa}^{\rm eff}$$

Reciprocity relation

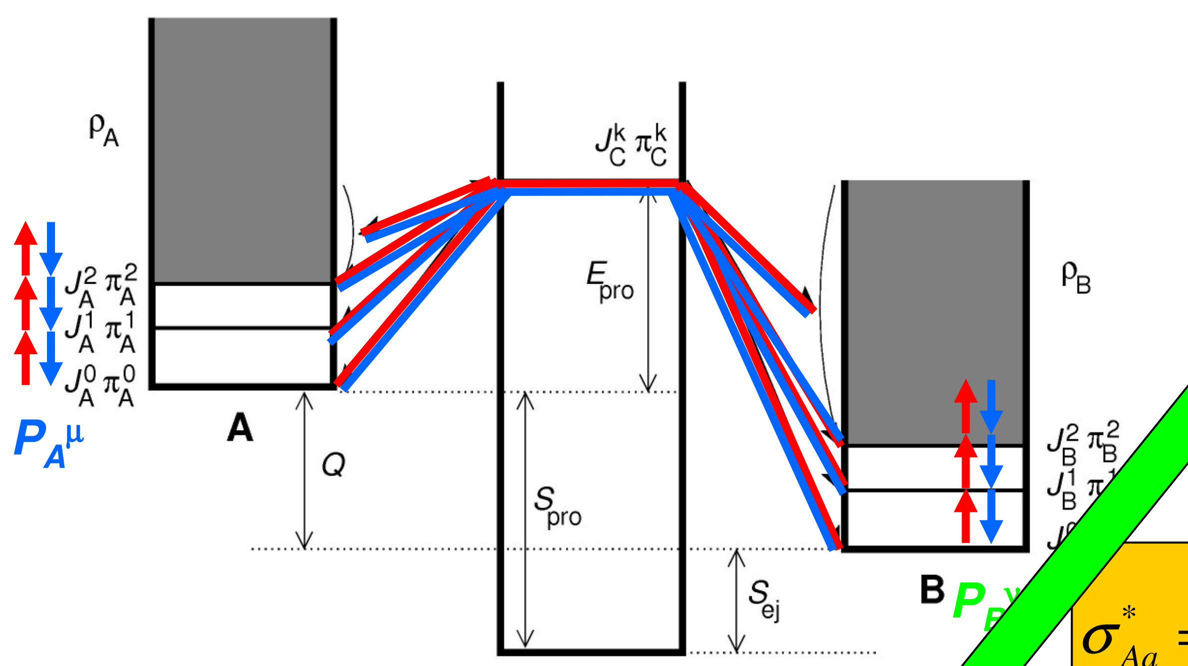
Step 2: Let's add thermal population of excited states → Detailed Balance

$$P_A^\mu = \frac{N_A^\mu}{N_A^{\rm tot}} = \frac{g_A^\mu e^{-\varepsilon_A^\mu/(kT)}}{\sum_\mu g_A^\mu e^{-\varepsilon_A^\mu/(kT)}} = \frac{g_A^\mu e^{-\varepsilon_A^\mu/(kT)}}{G_A(T)} = \frac{g_A^\mu e^{-\varepsilon_A^\mu/(kT)}}{g_A^0 G_A^{\rm norm}}$$

From Saha equation; *G*(*T*) is partition function

$$\sigma_{Aa}^* = \sum_\mu P_A^\mu \sigma_{Aa}^\mu$$

Stellar cross section



Step 3: Insert in stellar rate

$$\sigma^{\text{eff}}_{Aa} = \sum_{\mu} \frac{g^{\mu}_{A}}{g^{0}_{A}} \frac{E^{\mu}_{A}}{E^{0}_{A}} \sigma^{\mu}_{Aa}$$

Effective c.s.

$$\sigma^{\text{eff}}_{Bb} E^{0}_{B} = \frac{g^{0}_{A} g_{a}}{g^{0}_{B} g_{b}} \frac{m_A}{m_B} E^{0}_{A} \sigma^{\text{eff}}_{Aa}$$

Reciprocity relation

$$\sigma^{*}_{Aa} = \sum_{\mu} P^{\mu}_{A} \sigma^{\mu}_{Aa} = \sum_{\mu} \frac{g^{\mu}_{A} e^{-\varepsilon^{\mu}_{A}/(kT)}}{g^{0}_{A} G^{\text{norm}}_{A}} \sigma^{\mu}_{Aa}$$

Stellar cross section

One MB distribution instead of many!

$$\langle \sigma v \rangle^{*}_{Aa} \propto \int \sigma^{*}_{Aa} E_A e^{-E_A/(kT)} dE_A = \frac{1}{G^{\text{norm}}_{A}} \int \left\{ \sum_{\mu} \frac{g^{\mu}_{A}}{g^{0}_{A}} \sigma^{\mu}_{Aa} E_A e^{-\left(E_A + \varepsilon^{\mu}_{A}\right)/(kT)} \right\} dE_A$$

$$= \ldots = \frac{1}{G^{\text{norm}}_{A}} \int \left\{ \sum_{\mu} \frac{g^{\mu}_{A}}{g^{0}_{A}} E^{\mu}_{A} \sigma^{\mu}_{Aa} e^{-E^{0}_{A}/(kT)} \right\} dE^{0}_{A} = \frac{1}{G^{\text{norm}}_{A}} \int \sigma^{\text{eff}}_{A} E^{0}_{A} e^{-E^{0}_{A}/(kT)} dE^{0}_{A}$$

Stellar rates obey reciprocity! This implies thermal equilibrium in BOTH nuclei A, B!

Simplification of Stellar Rate

MB distributed projectiles act on every excited state, have to do a weighted sum:

$$\langle \sigma v \rangle^*_{Aa} \propto \frac{1}{G_A^{\mathrm{norm}}} \sum_\mu \left(\int \left\{ \frac{g_A^\mu}{g_A^0} \sigma_{Aa}^\mu E_A^\mu e^{-\left(E_A^\mu + \varepsilon_A^\mu\right)/(kT)} \right\} dE_A^\mu \right)$$

$$= \ldots = \frac{1}{G_A^{\mathrm{norm}}} \int \left\{ \sum_\mu \frac{g_A^\mu}{g_A^0} E_A^\mu \sigma_{Aa}^\mu e^{-E_A^0/(kT)} \right\} dE_A^0 = \frac{1}{G_A^{\mathrm{norm}}} \int \sigma_A^{\mathrm{eff}} E_A^0 e^{-E_A^0/(kT)} dE_A^0$$

with effective cross section

$$\sigma_{Aa}^{\mathrm{eff}} = \sum_\mu \sum_\nu \frac{g_A^\mu}{g_A^0} \frac{E_A^\mu}{E_A^0} \sigma_{Aa}^{\mu\nu}$$

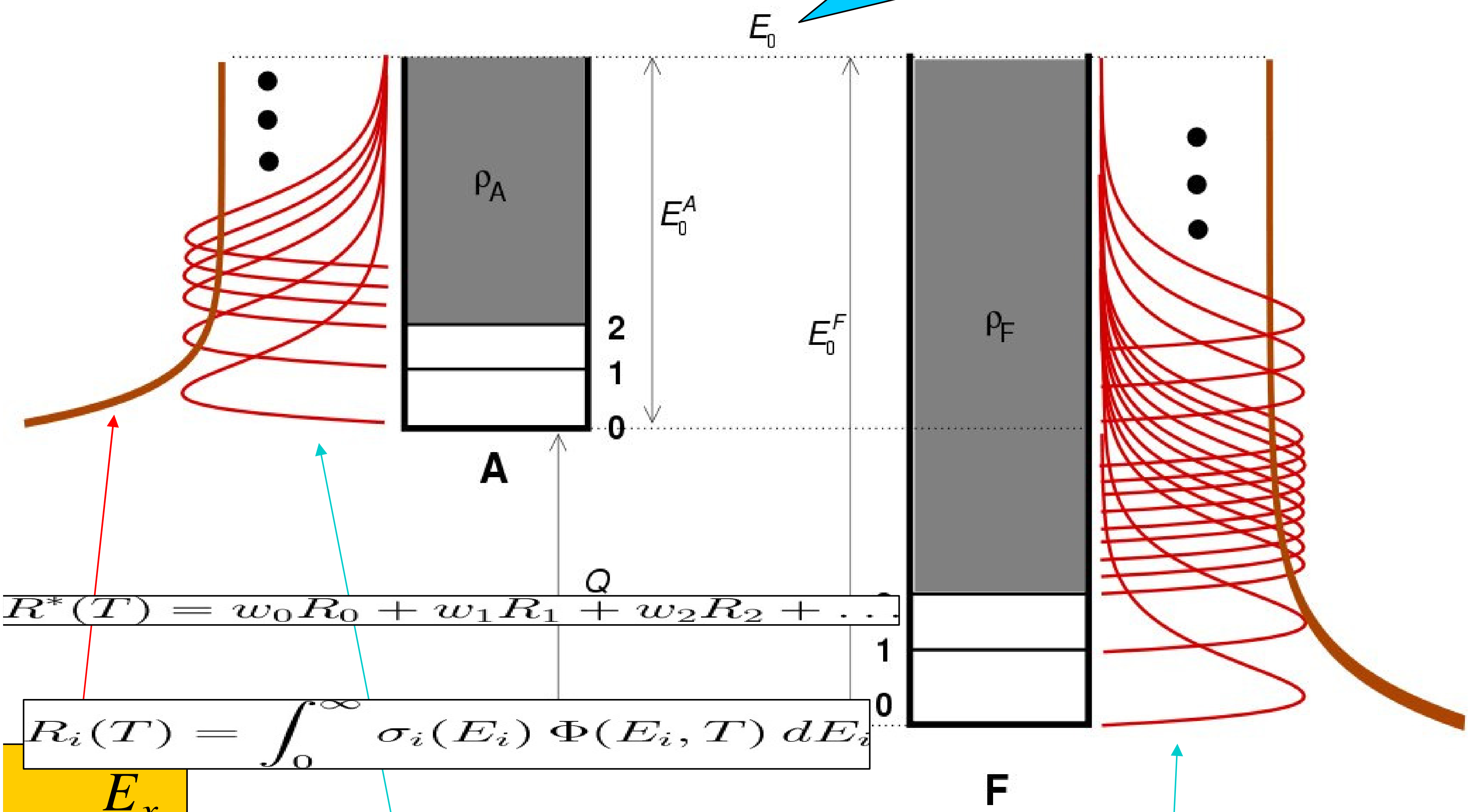
Effective cross section sums over all accessible excited states μ, ν in initial and final nucleus!

$g = 2J + 1$

$G^{\mathrm{norm}} \ldots$ normalized partition function

Effective weights … states

Gamow energy

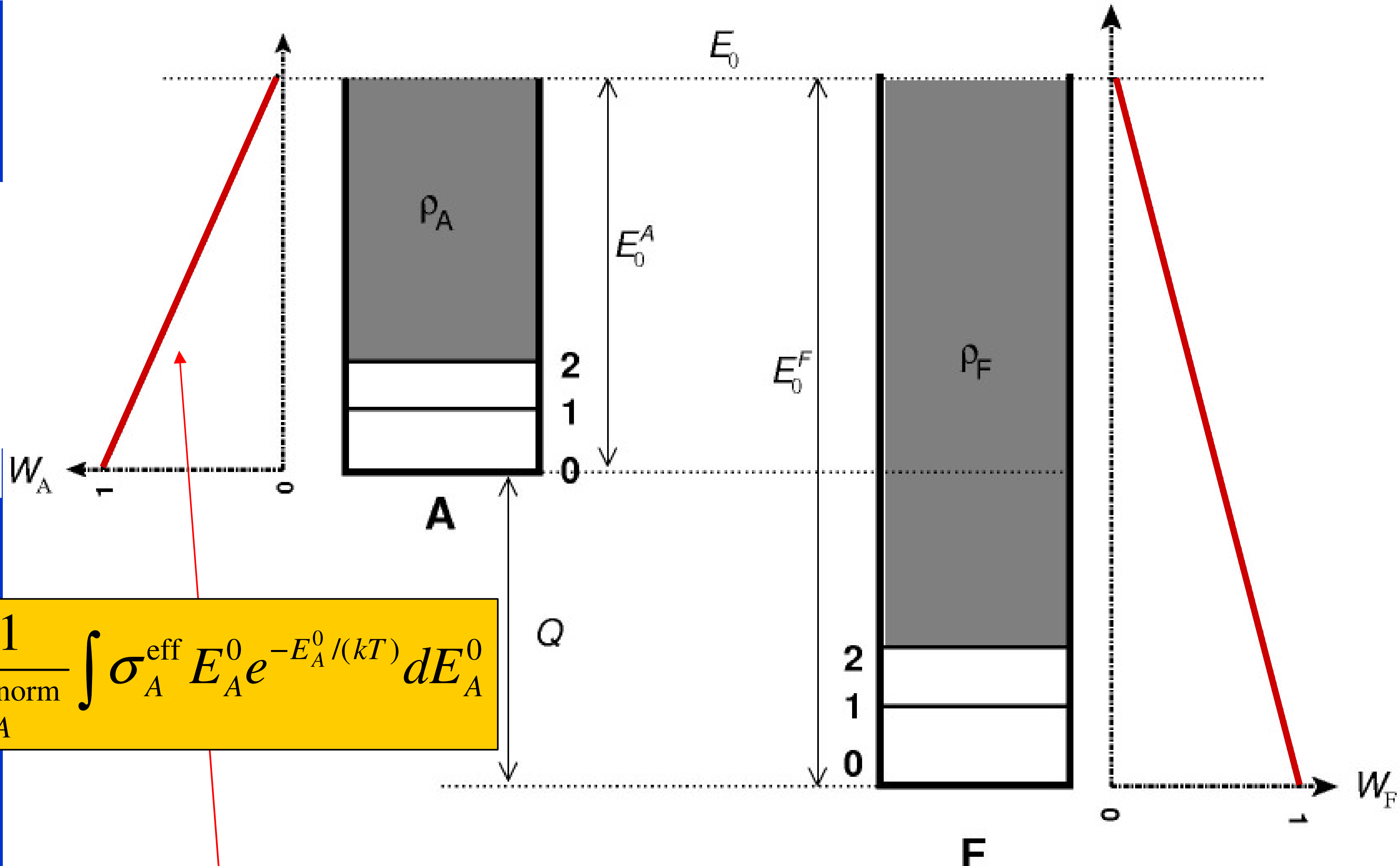


$$R^*(T) = w_0 R_0 + w_1 R_1 + w_2 R_2 + \ldots$$

$$R_i(T) = \int_0^\infty \sigma_i(E_i)\, \Phi(E_i, T)\, dE_i$$

$$e^{-\frac{E_x}{kT}}$$

MB distributions acting on nuclei in excited states



$$\frac{1}{G_A^{\mathrm{norm}}}\int \sigma_A^{\mathrm{eff}} E_A^0 e^{-E_A^0/(kT)}\, dE_A^0$$

$$e^{-\frac{E_x}{kT}} \rightarrow 1-\frac{E_x}{E_0}$$

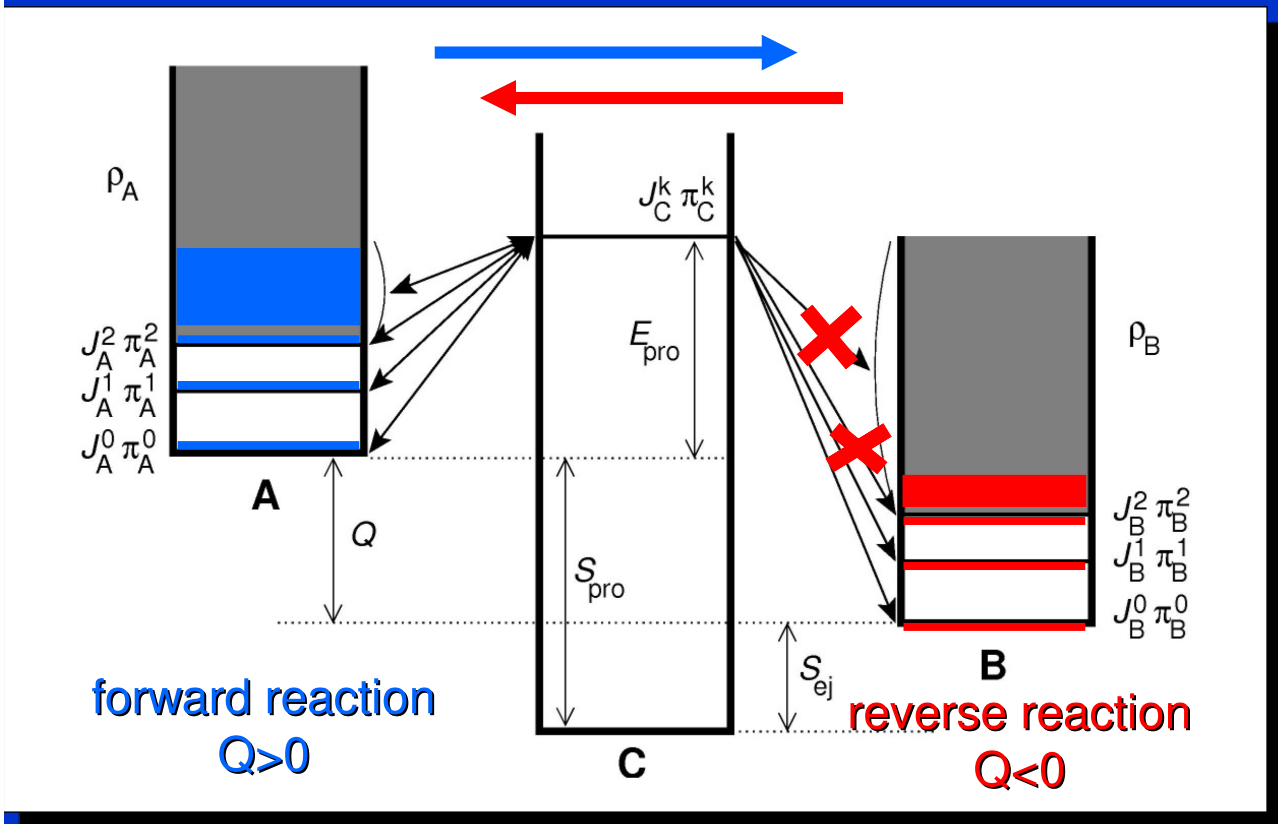
Always determine rate in direction of positive Q_{Aa}, to minimize SEF and numerical errors. For numerical stability in reaction networks, forward and backward rates have to be computed from ONE source!

Exceptions to the Q-value rule

(Coulomb) Enhancement of g.s. Contribution to Stellar Rate

- Exception to the rule of positive Q-value:
  - **Rauscher et al, PRC 80 (2009) 035801**
  - Kiss, Rauscher, Gyurky et al, PRL 101 (2008) 191101
- Interesting effect: Transitions on excited states can be suppressed differently in the entrance and exit channel
- This can lead to an inversion of the rule in some cases

Coulomb suppression of stellar enhancement (Coulomb enhancement of g.s. contribution)



It is usually assumed that $X_{forw} > X_{rev}$ and therefore a measurement of the forward reaction will be closer to stellar cross section.

However, low energy transitions of charged particles will be suppressed even when they are favored by spin selection. Thus, for reactions with different Coulomb barriers in the channels, an inversion is possible!

g.s. contribution: $X = \frac{r^{\mathrm{lab}}}{G_0 r^*}$

MB population: $P_i = (2J_i + 1)\,\mathrm{e}^{-\frac{E_i}{kT}}$

transition probability: T_i

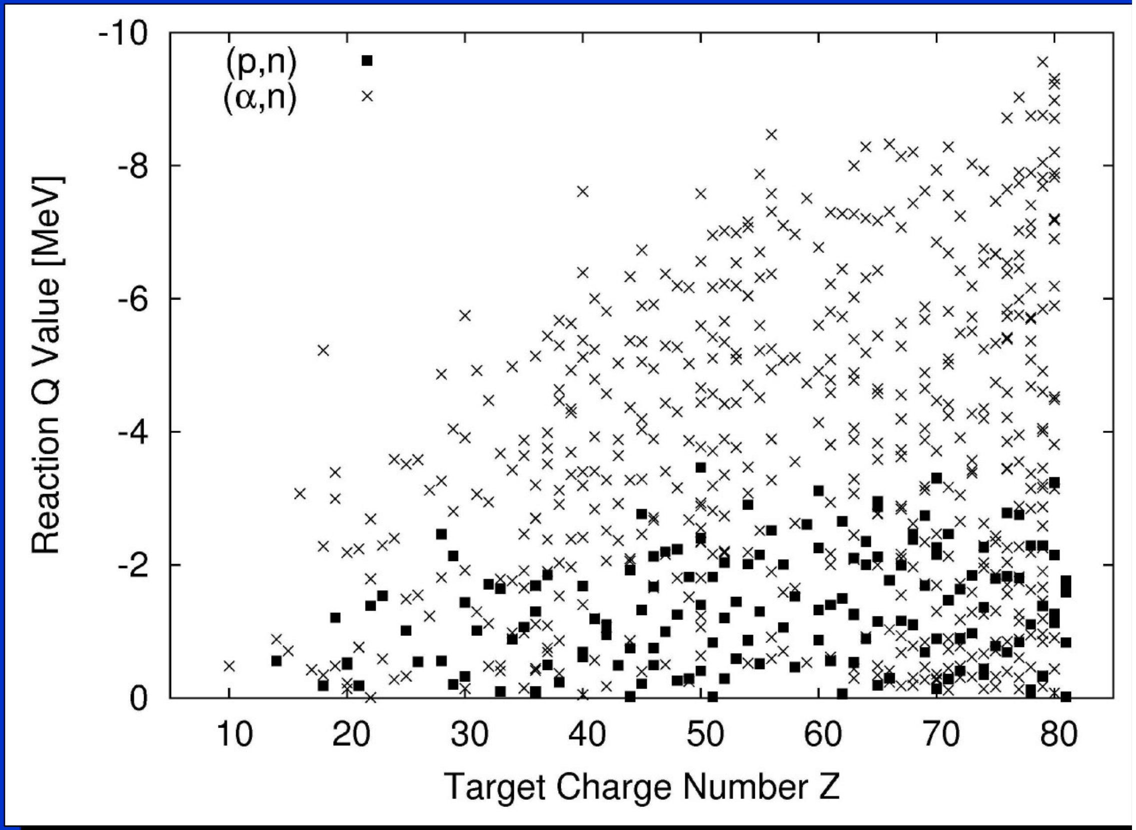
$$X_{\mathrm{forw}} < X_{\mathrm{rev}}$$

Coulomb suppression of stellar enhancement II

Prerequisite: Q value is sufficiently small with respect to Coulomb barrier in order to have only few transitions from excited states.
Example: Plot Q values of (p,n) and (α,n) reactions with $f_{forw}>f_{rev}$ and $f_{rev}\approx 1$:

When considering all kinds of reactions between proton and neutron drip from Ne to Bi, >1200 reactions of this type are found!

(not only (p,n), (α,n))



Selection of "interesting cases":

- $T < 4.5$ GK
- $f_{forw}/f_{rev} > 1.1$
- $f_{rev} < 1.5$

Phys. Rev. Lett. 101 (2008) 191101

Phys. Rev. C 80 (2009) 035801

Photodisintegrations

Photodisintegration and the γ-Process

- The γ-process derives its name from the importance of (γ,n), (γ,p), (γ,α) reactions
- But stellar photodisintegration rates are different from laboratory photodisintegration
- Not just because of thermal photon distribution but more so due to thermal excitation: the Q-value rule!
- Can be calculated from capture with reciprocity formula!

Connection to capture rate by <u>detailed balance</u>:

$$\lambda_{m\gamma} = \left(\frac{A_i A_j}{A_m}\right)^{3/2} \frac{(2J_i+1)(2J_j+1)}{2J_m+1} \frac{G_i^{\mathrm{norm}}(T)}{G_m^{\mathrm{norm}}(T)} \left(\frac{\mu kT}{2\pi\hbar^2}\right)^{3/2} \mathrm{e}^{-Q_{ij}/kT} \left\langle \sigma^* v \right\rangle_{ij}$$

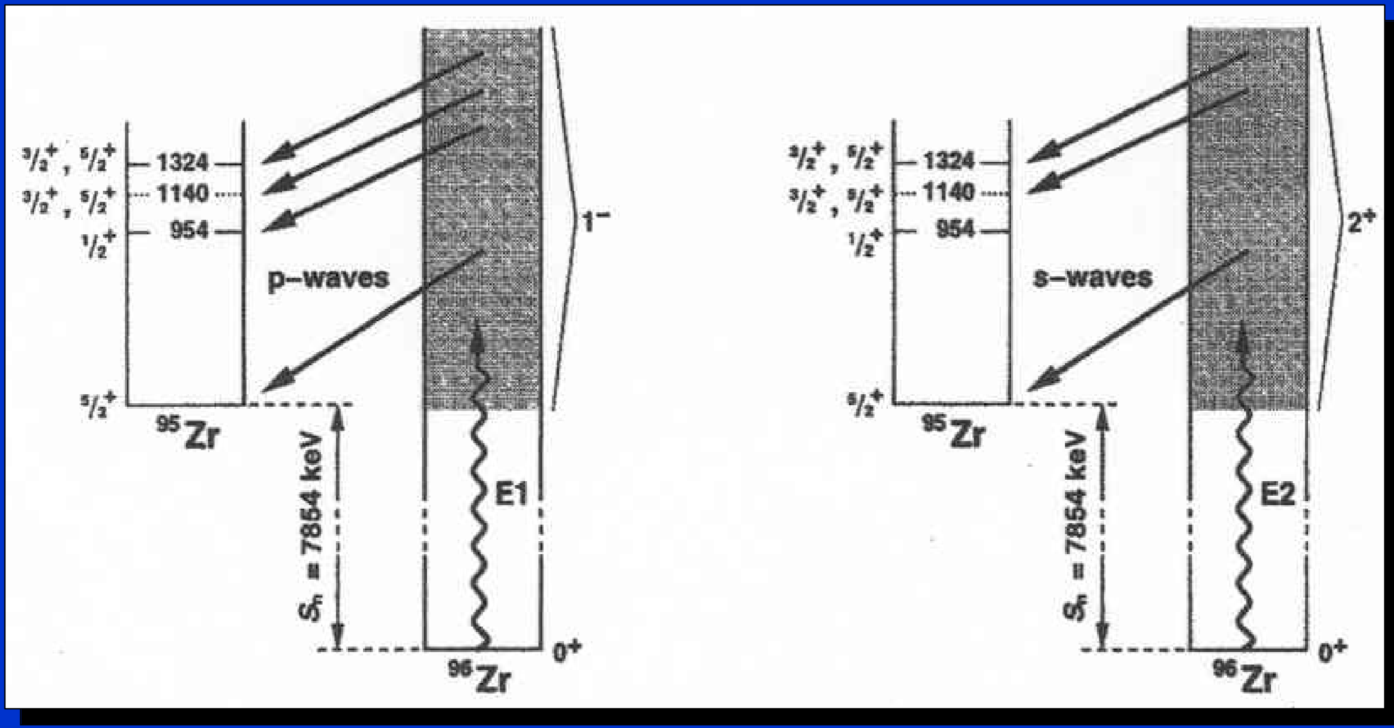
g.s. Contributions in Stellar Photodisintegration Rates

Target	$(\gamma$,n) g.s contribution (T_9=2.5)
^{86}Sr	0.00059
^{90}Zr	0.00034
^{96}Zr	0.0061
^{94}Mo	0.0043
^{142}Nd	0.0028
^{155}Gd	0.0012
^{186}W	0.00049
^{185}Re	0.00021
^{187}Re	0.00024

Target	$(\gamma$,n) g.s contribution (T_9=2.5)
^{186}Os	0.00016
^{190}Pt	0.000069
^{192}Pt	0.00011
^{198}Pt	0.0018
^{197}Au	0.00035
^{196}Hg	0.00043
^{198}Hg	0.00084
^{204}Hg	0.0088
^{204}Pb	0.0059

Simulating Photodisintegration

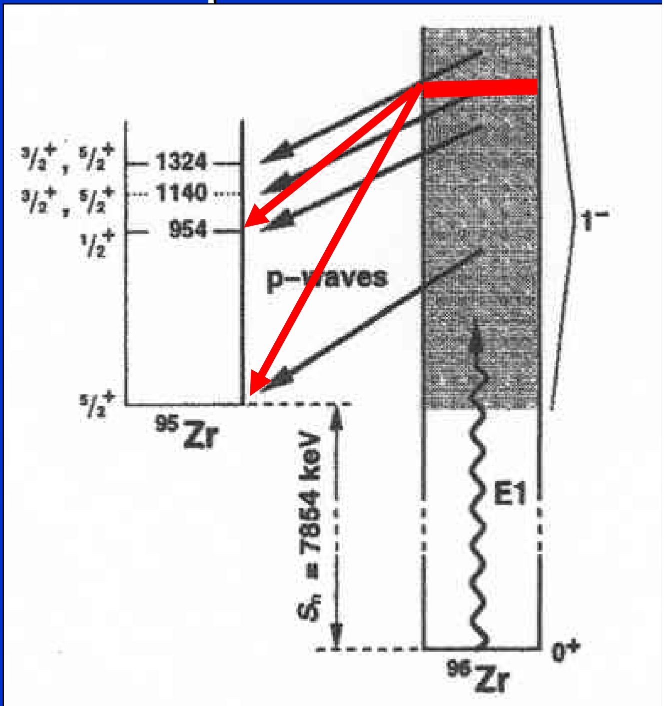
- Bremsstrahlung spectra or mono-energetic
- Simulate Photon-Bath by superposition
  - Can only probe ground-state transition: unrealistic rate
- Tests only few transitions!



Mohr et al. 2001, Vogt et al. 2003, Sonnabend et al. 2003

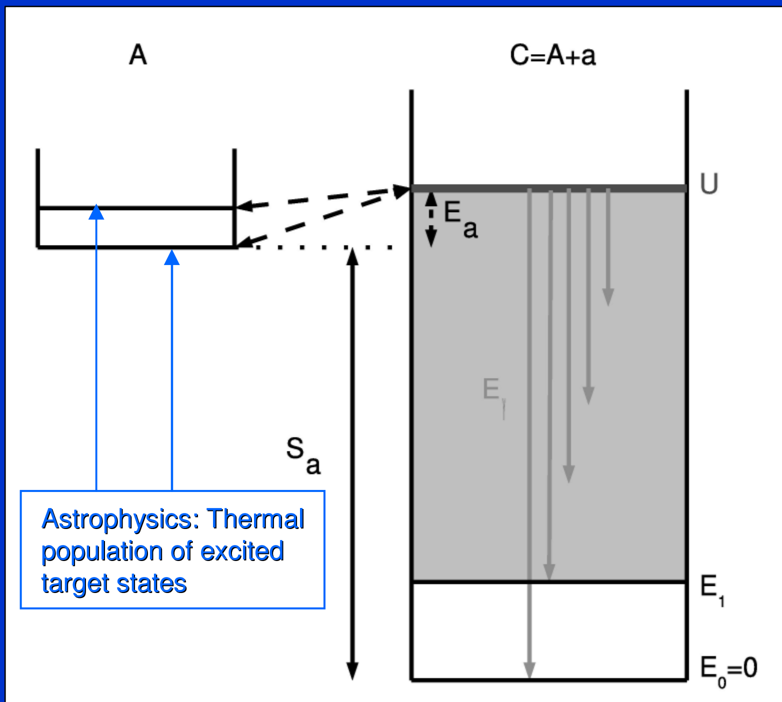
Simulating Photodisintegration II

- Maybe it can help to constrain (averaged) particle widths (optical potentials)?
  - except at the threshold, laboratory c.s. of (γ,x) reactions are mainly sensitive to γ-width! (unfolding required)
- So we learn something about the γ-width?
  - Yes, but only at „one“ energy
  - Astrophysically a large number of γ-transitions with smaller energy contributing
- Remaining possibility: check energy dependence of optical potentials
  - e.g., (γ,n_1)/(γ,n_0) ratios, γ-dependence cancels out
  - must be separated from spin/parity selection
  - different for each nucleus; needs theory support
  - perhaps (n,n‘) better suited? Further ideas?



Relevant γ-energies

Transition energies contributing to capture

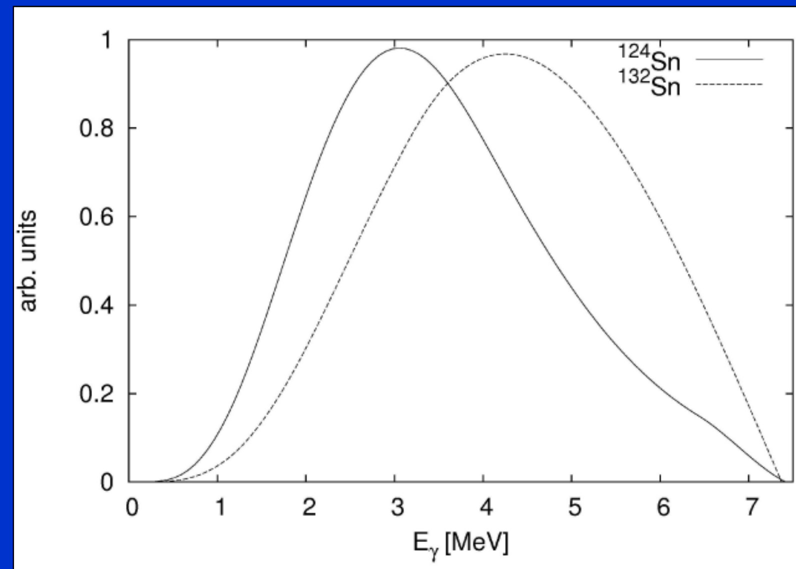


Competition between level density increase and decrease of transition strength:

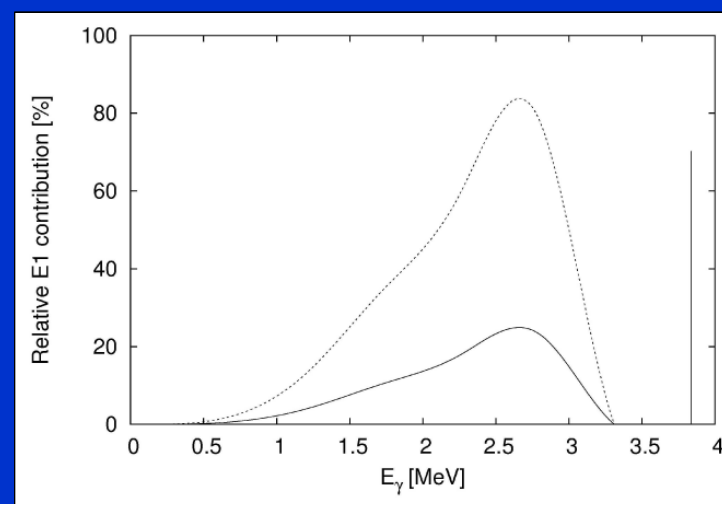
$$\rho \propto \frac{e^{2\sqrt{aU}}}{U^{5/3}}$$

$$T_{E1} \propto \frac{\Gamma_{GDR} E_\gamma^{\ 4}}{\left(E_\gamma^{\ 2} - E_{GDR}^{\ \ 2}\right)^2 + \Gamma_{GDR}^{\ \ 2} E_\gamma^{\ 2}}$$

$$T_{M1} \propto E_\gamma^{\ 3}$$

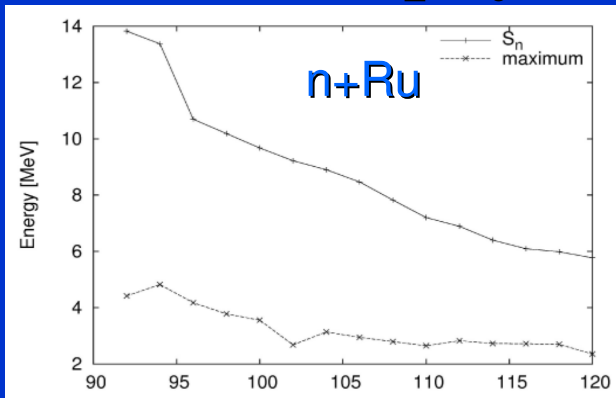


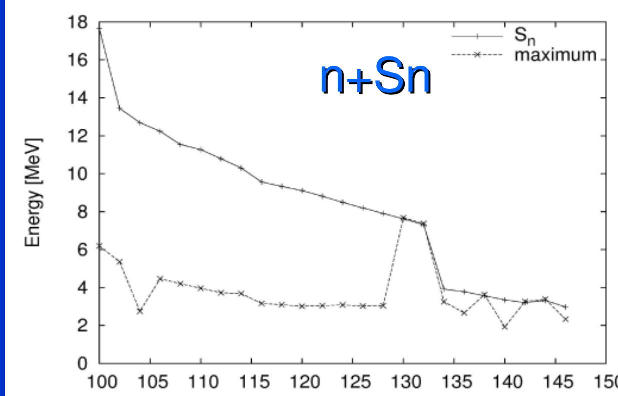
Transition to g.s. or isolated excited states often suppressed by selection rules:

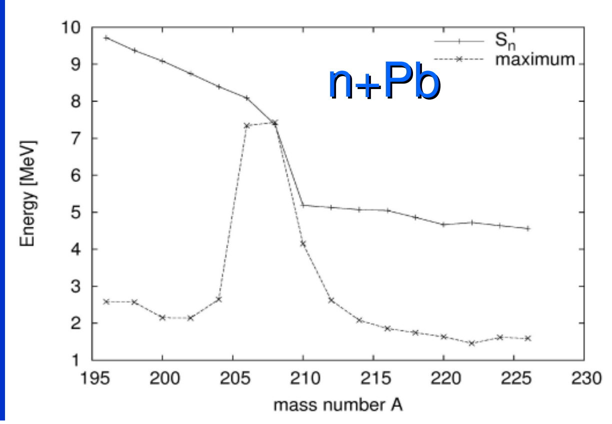


Rauscher, PRC 78 (2008) 032801(R)

Location of maximum contribution at astrophysically relevant reaction energies



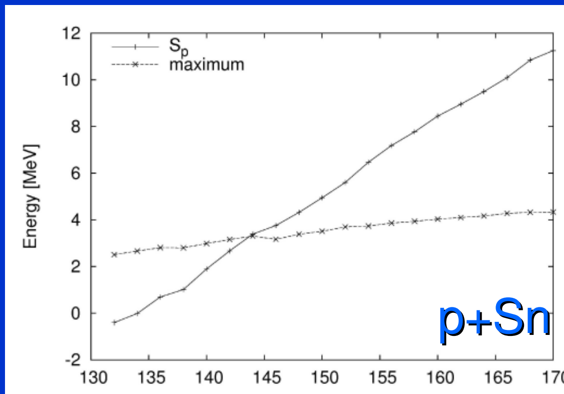


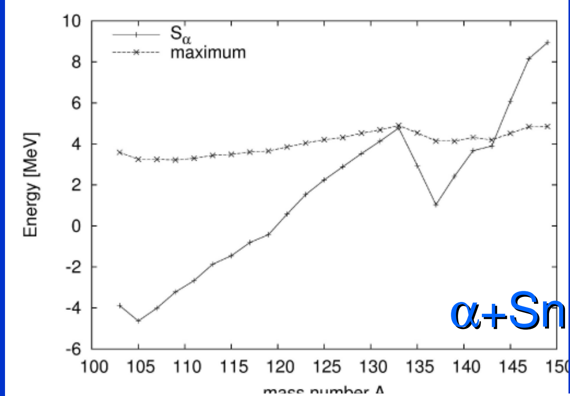


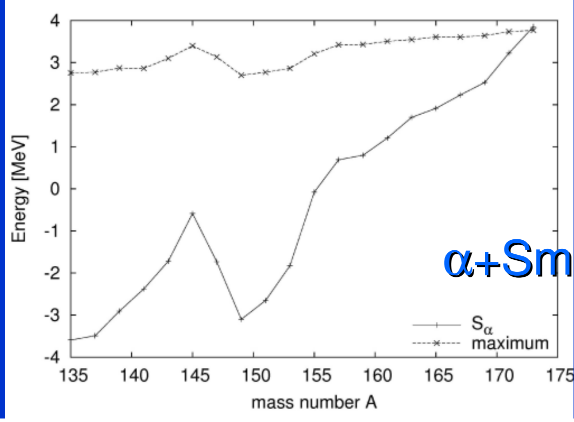
- Maxima located at 2-4 MeV
- quite independent of reaction
- Exception: nuclei with low level density (magic numbers or close to drip) → maximum shifted to higher energies (isolated states)
- Hauser-Feshbach not valid for exceptions

Important to judge relevance of modification of γ transition strength (e.g. pygmy resonance)

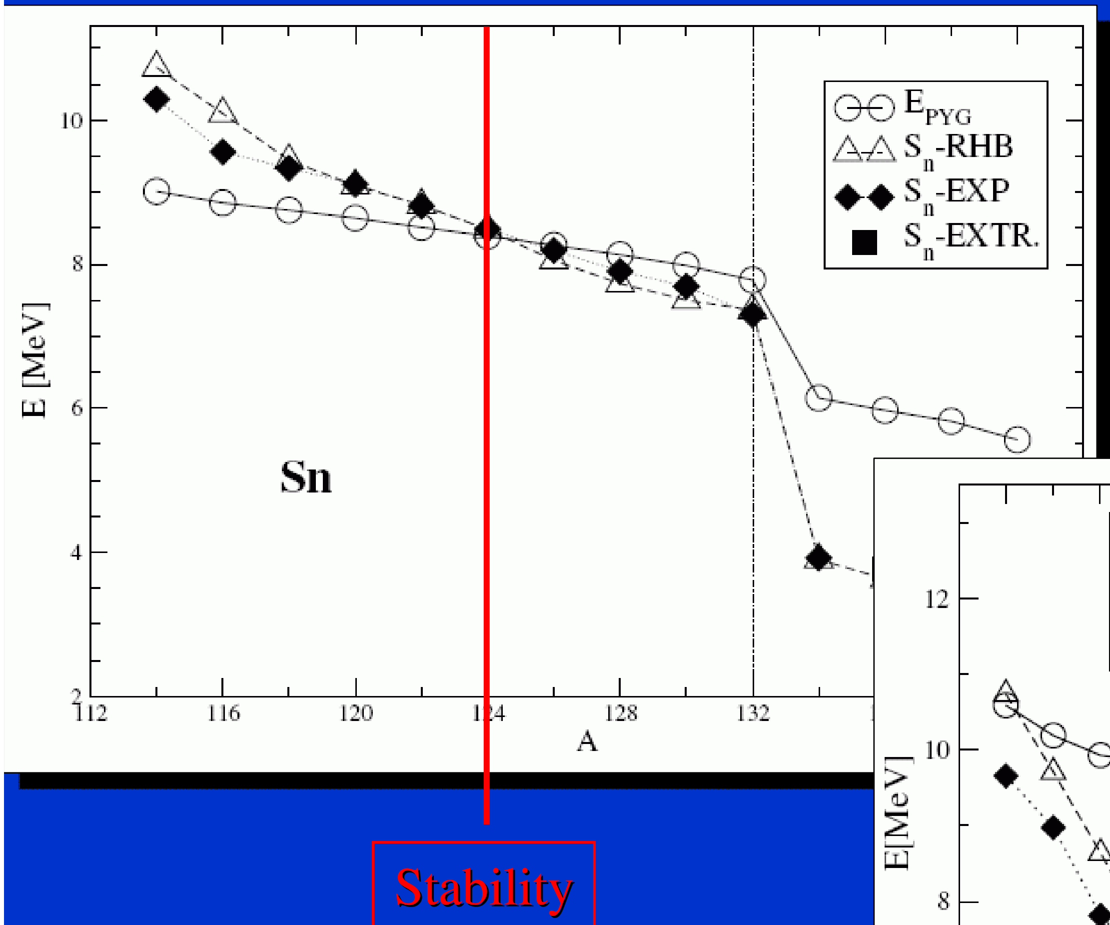
Rauscher, PRC 78 (2008) 032801(R)

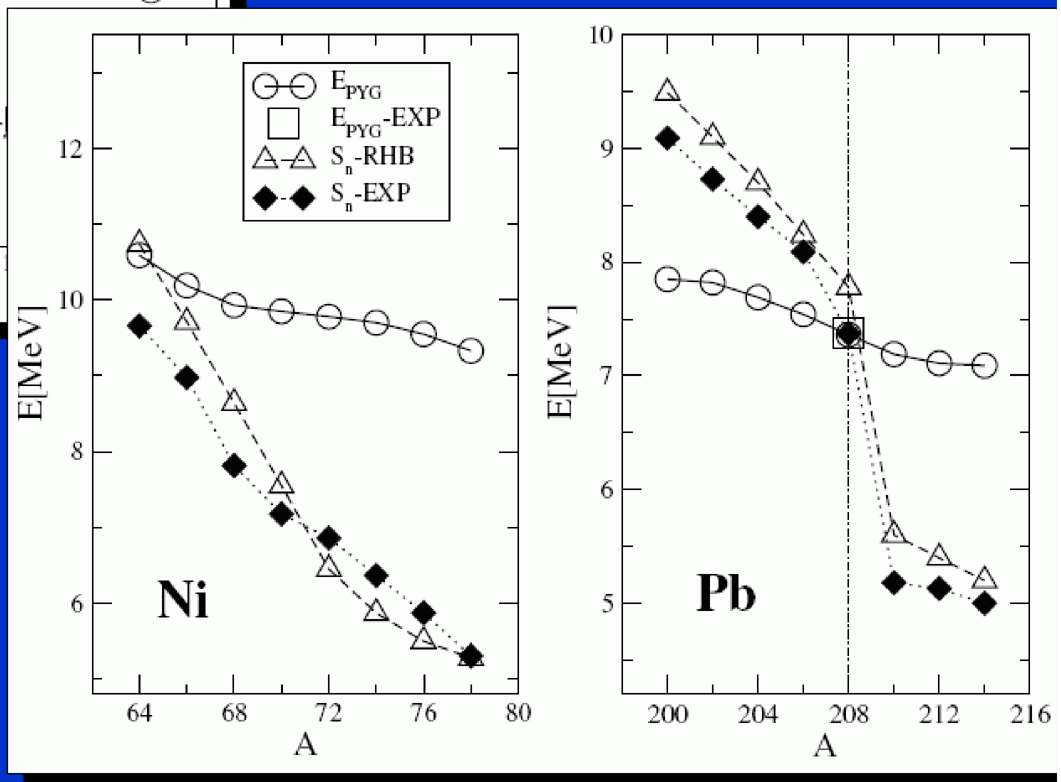






Pygmy Predictions



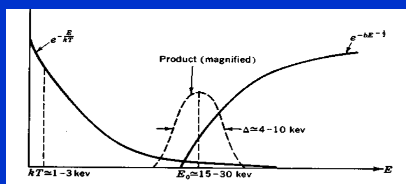


Paar, Nikšić, Vretenar, Ring 2005

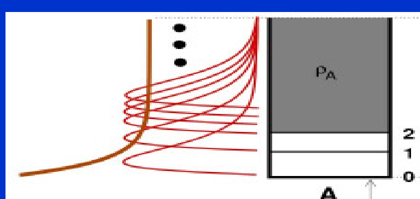
Summary and Conclusions

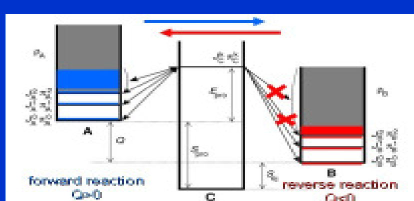
Summary I: Reaction Rates involving intermediate to heavy target nuclei

When assessing impact of nuclear physics or planning experiments, pay attention to:



$$\sigma \propto \frac{\langle T_{\text{entrance}} \rangle \langle T_{\text{exit}} \rangle}{\langle T_{\text{total}} \rangle}$$





- Relevant energy range!
  - simple Gamow peak formula NOT correct!
  - *incorrect in text books*
- Sensitivities
  - different at astro energies, γ-width not always smallest
- Stellar modification of the rates
  - Many additional transitions from excited states!
  - NOT simple Boltzmann factor!
  - *incorrect in text books*
- *Nice new effect:* Coulomb suppression of stellar transitions
  - exception to the Q-value rule

Review: Rauscher, Int. J. Mod. Phys. E 20, 1071 (2011)

Summary II

Astrophysics cases:

- s-Process: (n,γ) rate not always well constrainable; γ-width also important
- γ-Process: (n,γ), (p,γ), (a,γ), (n,p); not well constrainable; required: optical potentials, γ-strength (neutron capture)

When picking cases to be studied in the laboratory:

1. Choose astrophysically relevant reaction
   - from literature or ask your favorite astrophysicist (caution: models may change…)
2. Check g.s. contribution
   - if small, check whether special transitions or properties may be studied, at least (see 5.)
3. Check astrophysically relevant energy range
4. Check experimental feasibility
5. Check rate and cross section sensitivities
   - if g.s. contribution small or measurement impossible in astro energy range, perhaps something can still be learned